

# Logic for the Friendship of Nations

January 14 – 15, 2022

Laura Crosilla

University of Oslo, Norway

Title: The infinite, between philosophy and logic

Abstract: I consider examples of profitable interaction between philosophy and logic, with special focus on the concept of infinity. My discussion will be inspired by fundamental philosophical reflections by prominent mathematicians, such as David Hilbert, Hermann Weyl, Solomon Feferman and Errett Bishop.

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Thomas Ågotnes

University of Bergen, Norway

Title: Reasoning about Group Knowledge

Abstract: Logic can be seen as describing the reasoning of an ideal reasoner. What if that reasoner reasons about the reasoning of another reasoner - who again reasons about the reasoning of the first reasoner, and so on? We get logic on steroids - logic about logic! We can take this even further: what about groups? Does it make sense to talk about logic on the level of groups - can groups reason in a logical consistent way? For example, if an (ideal) group knows that  $P$  and that  $P$  implies  $Q$ , does it then necessarily know that  $Q$ ? If it does not know that  $P$ , does it know that it does not know that  $P$ ? Answers to these questions are ultimately relevant for consensus building and information aggregation across groups, communities and cultures. I will address them on one particular level of abstraction, using what is called (modal) epistemic logic - logic for reasoning about knowledge. This is a semi-popular talk that should (hopefully) be accessible to a general audience.

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Martin Ziegler

KAIST and the Korean Association for Mathematical Logic, Republic of Korea

Title: Logic of Deterrence?

Abstract: Deterrence Theory claims to apply Game Theory to practical political situations. John von Neumann, Thomas Schelling, and Hermann Kahn had developed and used it to advise their government. Proponents credit Deterrence Theory with ensuring Peace during the Cold War, while opponents claim that Peace was maintained only IN SPITE of mutual threats and refer for example to Stanislav Petrov DEVIATING from instructions. Such ambiguity, similar to Euathlus' Paradox of the Court, hints at an intrinsic logical contradiction. We recall Bertrand Russel's life between Logic and Peace. We then proceed to a layperson's introduction to Formal Logic, to paradoxes, contradictions, and consistency. And we conclude with the question (but not answer) of whether Deterrence Theory can be put on a consistent logical foundation.

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Layth Atheer Yousef

Al-Mustansiriya University, Iraq

Title: Yasin Khalil ,the distinguished Iraqi Logician.

Abstract: TBA

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Mohua Banerjee

Indian Institute of Technology Kanpur, India

Title: A journey through algebras of rough sets and their logics

Abstract: Algebraic studies related to rough set theory began soon after the theory was proposed by Pawlak in 1982. In this talk, we give a glimpse of our work on the subject that focusses on some algebras that rough sets form, and the logics corresponding to these algebras. Different operations on rough sets lead to the formation of different algebraic structures – some of which are instances of well-known classes of algebras (e.g. Kleene algebras), while others define new algebras (e.g. topological quasi-Boolean algebras). Some of these classes of algebras turn out to be representable by the ones formed by rough sets. As a consequence of the representation results, the logics corresponding to the algebras get a rough set semantics.

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S. P. Suresh

Chennai Mathematical Institute, India

Title: Inference in Nyaya

Abstract: Nyaya shastra, the traditional study of logic in India, has had a history of at least two millenia. One of the six orthodox philosophical systems, it is valued for its sophisticated epistemology, which is adopted to various degrees by other systems too. Of particular interest is its focus on inference as a means of knowledge. In this talk, we present the basics of inference in Nyaya, with particular emphasis on the notion of vyapti (which can be roughly translated as logical implication). We will present traditional examples, as well as some more relatable modern examples to explain the concepts. The material for the talk is based on two traditional primers on Nyaya in Sanskrit.

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Jamshid Derakhshan

University of Oxford, United Kingdom

Title: On some applications of mathematical logic in algebra and number theory

Abstract: Many developments in model theory have been inspired by mathematical questions. These have led to powerful methods in logic as well as applications in different areas of mathematics. A very influential question was Hilbert's tenth problem (1900) on decidability of integer solvability of systems of polynomial equations over the integers. While the answer was negative, decidability turned out to hold (in the more general context of the first-order theory) for the real numbers and the complex numbers (Tarski, 1931), for the class of all finite fields (Ax, 1968) and for  $p$ -adic numbers (Ax and Kochen, 1965). These results have turned out to have important consequences for various mathematical structures. More refined results on quantifier elimination for  $p$ -adic fields by Macintyre (1976) and others led to several advances in algebra, number theory and geometry by Basarab, Denef, Loeser and others. More recently another approach to such topics was given by Hrushovski and Kazhdan (2006). An essential role in these works is played by a theory of measure and integration in Henselian valued fields. I will discuss recent developments continuing and applying such results as follows. In the last 15 years, Macintyre and myself have developed a model theory for the ring of adèles of a number field, a structure of fundamental importance in modern number theory. This has opened up new avenues on connections of logic to number theory. We develop and use a model theory for "restricted" products of structures building on foundational work of Feferman and Vaught (1959). An application of our work is a recent solution to a question of Ax from 1968 on decidability of the class of all the finite rings which are quotient rings of the ring of integers. Other applications include rigidity results in adèle rings on isomorphism and elementary equivalence, and results on axiom systems. Since the 1980's, Grunewald, Segal, Smith, du Sautoy, Lubotzky and others have developed the subject of subgroup growth that studies asymptotic questions in group theory using analytic properties of zeta functions attached to groups. Continuing this line with use of model-theoretic tools one can obtain new results on algebraic groups over  $p$ -adic fields (joint work with Berman, Onn, and Pajaanen, 2014) and over number fields (myself, 2022). Further connections and applications to several questions in number theory and Diophantine geometry are emerging, as well as challenging open problems.

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Larry Moss

Indiana University Bloomington, United States

Title: A Place for Logic in the Computer Processing of Language

Abstract: Starting in 2018, computers have been able to carry out some tasks at human level (or better), tasks which are traditionally thought of as 'logical'. These include the central task of logic: knowing 'what follows from what', when everything is presented in natural language. We therefore are at a watershed moment in the history of logic. However, the computational systems -- neural net learners -- do not use logic in any evident manner. Of course logic is involved in computer science at many levels, but the particular programs involved in inference are much more like the ones that memorize patterns and classify objects. They do not use explicit symbolic reasoning of the kind logicians love.

This talk is concerned with attempts by several groups of researchers to do reasoning in language on the computer, or to probe the deep learners to see how much they really can do, or to create hybrid symbolic/neural reasoning systems. I will try hard to speak to a general audience.

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Norbert Preining

Fujitsu Research Labs

Title: Gödel Logics - a short survey

Abstract: Gödel logics can be characterized in a rough-and-ready way as follows: The language is a standard (propositional, quantified-propositional, first-order) language. The logics are many-valued, and the sets of truth values considered are (closed) subsets of  $[0,1]$  which contain both 0 and 1. 1 is the 'designated value,' i.e., a formula is valid if it receives the value 1 in every interpretation. The truth functions of conjunction and disjunction are minimum and maximum, respectively, and in the first-order case quantifiers are defined by infimum and supremum over subsets of the set of truth values. The characteristic operator of Gödel logics, the Gödel conditional, is defined by  $A \rightarrow B = 1$  if  $A \leq B$ , and  $B$  otherwise. Because the truth values are ordered, the semantics of Gödel logics is suitable for formalizing comparisons. It is related in this respect to a more widely known many-valued logic, Lukasiewicz (or 'fuzzy') logic. In contrast to Lukasiewicz logic, which might be considered a logic of "absolute" or "metric comparison", Gödel logics are logics of "relative comparison." Approaching from a different angle, Gödel logic is one of the three basic t-norm based logics which have received increasing attention in the last 15 or so years (the others are Lukasiewicz and product logic). Yet Gödel logic is also closely related to intuitionistic logic: it is the logic of linearly-ordered Heyting algebras. In the propositional case, infinite-valued Gödel logic can be axiomatized by the intuitionistic propositional calculus extended by the axiom schema  $(A \rightarrow B) \vee (B \rightarrow A)$ . This connection extends also to Kripke semantics for intuitionistic logic: Gödel logics can also be characterized as logics of (classes of) linearly ordered and countable intuitionistic Kripke structures with constant domains. Furthermore, the infinitely valued propositional Gödel logic can be embedded into the box fragment of LTL in the same way as intuitionistic propositional logic can be embedded into  $S4$ . We will give a short overview of this family, presenting propositional, quantified propositional (weak second order), and first order logics, including proof-theory, relation to intuitionistic logic, and recent results on fragments.

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Koji Nakatogawa

Hokkaido University, Japan

Title: Cantor's paradise, Vairocana, and Tarski-Grothendieck-Lawvere Semantics

Abstract: E.Zermelo introduced a strongly inaccessible cardinal number (originally called *Glent Zahlen* by E.Zermelo) in 1931. He called the domain of sets constructed from the empty set by the repeated applications of powerset operation as the modernized version of Cantor's universe. In G.Cantor's theology-oriented paper, above the strata of mathematical infinities, there lies the next strata of the created nature (what is created by God?). The strata of angels comes next above, and then the God. Large finite numbers discussed in *Avantamska Sutra* have already been investigated by scholars. Infinities in the *Avantamska Sutra* seem not to be well researched. They are described by expressions such as river, rain, wind, cloud, mountain, and the sea. The Ganges (Ganges River) is used as an exemplification of a relatively small size of infinity (whether it may be its length or the number of its sand grains). It would be well worth asking whether E.Zermelo's *Glents Zahlen* correspond to the infinite size represented by the Ganges, or to the infinite size of the sea. Also, trees in an illusory garden described in *Avantamska sutra* might turn out to have the structure of an *Aronszejin tree* (hopefully that of  $\aleph_2$  *Aronszejin tree*) As the methodological framework of our considerations, not only Tarski-Vaught semantics based on set theory, but also Grothendieck-Lawvere semantics based on category theory will be employed. One should take note of what lies at the foundation of F.W.Lawvere's semantics. It is "the things in their motions", not "thing in itself (*Ding an Sich*) which is located outside of Time and Space. Philosophy of Buddhism advocates 'everything flows'.

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Athipat Thamrongthanyalak

Chulalongkorn University, Thailand

Title: Trace problems in tame expansions of the real field.

Abstract: Let  $F$  be a set of maps from  $\mathbb{R}^n$  to  $\mathbb{R}^m$  and  $S \subseteq \mathbb{R}^n$ . The trace of  $F$  to  $S$  is  $F|_S := \{f|_S : f \in F\}$ . Trace problems, which are important questions in analysis and topology, study the following question: What is a complete description of the trace of  $F$  to  $S$ ? When  $F$  is the space of continuous maps from  $\mathbb{R}^n$  to  $\mathbb{R}^m$ , this problem is related to Tietze's Extension Theorem. Roughly, we are interested in trace problems when  $F$  is a space of maps that satisfy certain nice properties. In this talk, we are interested in trace problems and its connection to definability in tame expansions of the real field.

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Zu Yao Teoh

Logic Association of Malaysia, Malaysia

Title: Logic in Malaysia, A Journey

**Abstract:** In this talk, I will take you through a journey of Mathematical Logic in Malaysia, the way I know it as well as my personal journey. It is in the hope that through this event, we, Logicians from around the world, will build a logical friendship and help one another to advance the education and research in Mathematical Logic. As such, I will also take the opportunity to introduce the Logic Association of Malaysia or, Persatuan Mantik Malaysia (PMM) as it is known locally, which was established in July last year, and speak of the near-term plans we have in line. I will also introduce some attractive places in Malaysia, to which we can one day hold Logic events and tourism at the same time

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Mohammad Mohsin Ebaish

Al-Mustansiriya University, Iraq

Title: The image of logician in Islamic culture

Abstract: TBA

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Nidal Zakir Azab

Al-Mustansiriya University, Iraq

Title: Analytical reading on the logical books of Islamic Philosophers

Abstracts: TBA

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Anuj Dawar

University of Cambridge, United Kingdom

Title: Complexity and the Expressive Power of Logics

Abstract: Much work in modern mathematical logic can be seen as the formal study of the interaction of three important elements in mathematics: language, structure and proof. Mathematical logic has found widespread application in computer science with different computational elements taking on these three roles. With a brief review of this background, I will look at the insight into computational complexity gained by understanding this complexity in terms of the expressive power of logical languages.

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Mirna Džamonja

IRIF (CNRS & Université de Paris), France

Title: Trees in set theory and contributions to the subject by the (ex-)Yugoslav mathematicians

Abstract: I was invited as a representative member of the rich logic community coming from the former Yugoslavia and hence have chosen to speak about one (of many) parts of logic where this community has been successful, namely set theory. Within set theory, I will illustrate the contributions by mentioning several results about the classical object of study, a tree. Set theory in the region of former Yugoslavia was started by Djuro Kurepa (1907-1993), a prolific mathematician who published more than 700 articles, was professor at the University of Zagreb and the University of Belgrade and academician. His most important work in set theory concerned various trees, notably Suslin trees and, evidently, Kurepa trees. His 1935 thesis at the Sorbonne with Fréchet is where one can find the now classical construction of an Aronszajn tree. He left a lasting legacy in set theory internationally, but also locally, since his influential book « Teorija skupova », on set theory, topology and measure theory was the book that everybody interested in set theory read. Kurepa proved that there exists a Suslin line iff there exists a Suslin tree and introduced the sigma-functor on trees. Kurepa's brilliant student Stevo Todorcevic is an academician and a world famous set theorist. Ever since his Master's thesis (1978) where he proved the consistency of MA with the negation of the weak Kurepa hypothesis, he continued to work with trees, giving (joint with Abraham) the construction of a rigid Aronszajn tree, the notion of Lipshitz trees, the non-existence of a universal Aronszajn tree. Another excellent set theorist from the Belgrade school is Boban Velickovic, who for example proved the consistency of a forcing axiom for the poset of all perfect trees together with the continuum large. I come from the University of Sarajevo, where set theory was brought by Karl Menger's student Harry Miller, who was a professor at the University of Sarajevo. My own work also involves trees, for example a joint result with Shelah that under MA there are not universal wide Aronszajn trees. This result is being extended to trees of size and height  $\aleph_2$  in our joint work with Rahman Mohammadpour, an Iranian mathematician who was a student of Boban. Speaking of students, we all had some really good students - for example Stevo's student Ilijas Farah is the world expert on applications of set theory to operator algebra. Another school of set theory in the region is in Novi Sad, where the senior member Milos Kurilic leads a considerably sized group. The logic community from former Yugoslavia is spread all over the world, but we have all kept close and friendly connections that have not been damaged by the unfortunate political events in the last 30 years.

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