7

Reinforced Concrete Structures

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7.1 Introduction

Reinforced concrete is a composite material. A lattice or cage of steel bars is embedded in a matrix of Portland cement concrete (see Figure 7.1). The specified compressive strength of the concrete typically ranges from 3,000 to 10,000 psi. The specified yield strength of the reinforcing steel is normally 60,000 psi. Reinforcement bar sizes range from $\frac{3}{8}$ to $2\frac{1}{4}$ in. in diameter (see Table 7.1). The steel reinforcement bars are manufactured with lugs or protrusion to ensure a strong bond between the steel and concrete for composite action. The placement location of the steel reinforcement within the concrete is specified by the concrete *cover*, which is the clear distance between the surface of the concrete and the reinforcement. Steel bars may be bent or hooked.

The construction of a reinforced concrete structural element requires molds or forms usually made of wood or steel supported on temporary shores or falsework (see Photo 7.1). The reinforcement bars are typically cut, bent, and wired together into a mat or cage before they are positioned into the forms. To maintain the specified clear cover, devices such as bar chairs or small blocks are used to support the rebars. Concrete placed into the forms must be vibrated well to remove air pockets. After placement, exposed concrete surfaces are toweled and finished, and sufficient time must be allowed for the concrete to set and cure to reach the desired strength.

The key structural design concept of reinforced concrete is the placement of steel in regions in the concrete where tension is expected. Although concrete is relatively strong in compression, it is weak in tension. Its tensile cracking strength is approximately 10% of its compressive strength. To overcome this weakness, steel reinforcement is used to resist tension; otherwise, the structure will crack excessively and may fail. This strategic combination of steel and concrete results in a composite material that has high strength and retains the versatility and economic advantages of concrete.

To construct concrete structures of even greater structural strength, very high-strength steel, such as Grade 270 strands, may be used instead of Grade 60 reinforcement bars. However, the high strength levels of Grade 270 steel is attained at high strain levels. Therefore, for this type of steel to work effectively with concrete, the high-strength strands must be prestrained or prestressed. This type of structure is

PHOTO 7.1 A 30-story reinforced concrete building under construction. The Pacific Park Plaza is one of the largest reinforced concrete structures in the San Francisco Bay area. It survived the October 17, 1989, Loma Prieta earthquake without damage. Instrumentation in the building recorded peak horizontal accelerations of 0.22*g* at the base and 0.39*g* at the top of the building (courtesy of Mr. James Tai, T.Y. International, San Francisco).

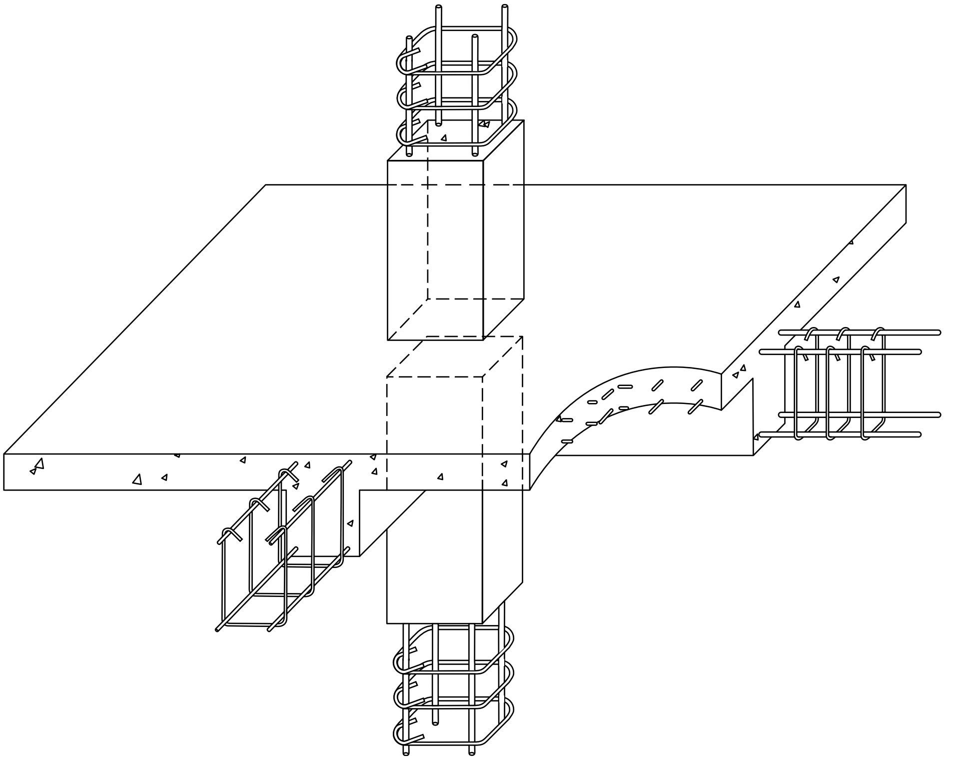

FIGURE 7.1 Reinforced concrete structure.

TABLE 7.1 Reinforcing Bar Properties

	Nominal properties		
Bar size	Diameter (in.)	Area (in.^{2})	Weight (lb/ft)
3	0.375	0.11	0.376
4	0.500	0.20	0.668
5	0.625	0.31	1.043
6	0.750	0.44	1.502
7	0.875	0.60	2.044
8	1.000	0.79	2.670
9	1.128	1.00	3.400
10	1.270	1.27	4.303
11	1.410	1.56	5.313
14	1.693	2.25	7.650
18	2.257	4.00	13.600

Note: Yield stress of ASTM 615 Grade 60 bar = 60,000 psi; modulus of elasticity of reinforcing steel = 29,000,000 psi.

referred to as *prestressed concrete*. Prestressed concrete is considered an extension of reinforced concrete, but it has many distinct features. It is not the subject of this chapter.

7.2 Design Codes

The primary design code for reinforced concrete structures in U.S. design practice is given by the American Concrete Institute (ACI) 318. The latest edition of this code is dated 2002 and is the main reference of this chapter. Most local and state jurisdictions, as well as many national organizations, have adopted ACI 318 for the coverage of reinforced concrete in their design codes. There may be minor changes or additions. The ACI code is incorporated into International Building Code (IBC), as well as the bridge design codes of the American Association of State Highway and Transportation Officials (AASHTO). The ACI Code is recognized internationally; design concepts and provision adopted by other countries are similar to those found in ACI 318.

7.3 Material Properties

With respect to structural design, the most important property of concrete that must be specified by the structural designer is the compressive strength. The typical compressive strength specified, f'_c, is one between 3000 and 8000 psi. For steel reinforcement, Grade 60 (American Society for Testing and Materials [ASTM] A615), with specified yield strength $f_y = 60{,}000$ psi, has become the industry standard in the United States and is widely available (see Photo 7.2). Material properties of concrete relevant for structural design practice are given in Table 7.2.

PHOTO 7.2 Installation of reinforcing bars in the Pacific Park Plaza building (courtesy of Mr. James Tai, T.Y. International, San Francisco).

TABLE 7.2 Concrete Properties

Concrete strength f'_c (psi)	Modulus of elasticity, $57{,}000\sqrt{f'_c}$ (psi)	Modulus of rupture, $7.5\sqrt{f'_c}$ (psi)	One-way, shear baseline, $2\sqrt{f'_c}$ (psi)	Two-way, shear baseline, $4\sqrt{f'_c}$ (psi)
3000	3,122,019	411	110	219
4000	3,604,997	474	126	253
5000	4,030,509	530	141	283
6000	4,415,201	581	155	310
7000	4,768,962	627	167	335
8000	5,098,235	671	179	358

Note: Typical range of normal-weight concrete = 145 to 155 pcf; typical range of lightweight concrete = 90 to 120 pcf.

7.4 Design Objectives

For reinforced concrete structures, the design objectives of the structural engineer typically consist of the following:

1. To configure a workable and economical structural system. This involves the selection of the appropriate structural types and laying out the locations and arrangement of structural elements such as columns and beams.
2. To select structural dimensions, depth and width, of individual members, and the concrete cover.
3. To determine the required reinforcement, both longitudinal and transverse.
4. Detailing of reinforcement such as development lengths, hooks, and bends.
5. To satisfy serviceability requirements such as deflections and crack widths.

7.5 Design Criteria

In achieving the design objectives, there are four general design criteria of SAFE that must be satisfied:

1. *Safety, strength, and stability.* Structural systems and member must be designed with sufficient margin of safety against failure.
2. *Aesthetics.* Aesthetics include such considerations as shape, geometrical proportions, symmetry, surface texture, and articulation. These are especially important for structures of high visibility such as signature buildings and bridges. The structural engineer must work in close coordination with planners, architects, other design professionals, and the affected community in guiding them on the structural and construction consequences of decisions derived from aesthetical considerations.
3. *Functional requirements.* A structure must always be designed to serve its intended function as specified by the project requirements. Constructability is a major part of the functional requirement. A structural design must be practical and economical to build.
4. *Economy.* Structures must be designed and built within the target budget of the project. For reinforced concrete structures, economical design is usually not achieved by minimizing the amount of concrete and reinforcement quantities. A large part of the construction cost are the costs of labor, formwork, and falsework. Therefore, designs that replicate member sizes and simplify reinforcement placement to result in easier and faster construction will usually result in being more economical than a design that achieves minimum material quantities.

7.6 Design Process

Reinforced concrete design is often an iterative trial-and-error process and involves the judgment of the designer. Every project is unique. The design process for reinforced concrete structures typically consists of the following steps:

1. Configure the structural system.
2. Determine design data: design loads, design criteria, and specifications. Specify material properties.
3. Make a first estimate of member sizes, for example, based on rule-of-thumb ratios for deflection control in addition to functional or aesthetic requirements.
4. Calculate member cross-sectional properties; perform structural analysis to obtain internal force demands: moment, axial force, shear force, and torsion. Review magnitudes of deflections.
5. Calculate the required longitudinal reinforcement based on moment and axial force demands. Calculate the required transverse reinforcement from the shear and torsional moment demands.

6. If members do not satisfy the SAFE criteria (see previous section), modify the design and make changes to steps 1 and 3.
7. Complete the detailed evaluation of member design to include additional load cases and combinations, and strength and serviceability requirements required by code and specifications.
8. Detail reinforcement. Develop design drawings, notes, and construction specifications.

7.7 Modeling of Reinforced Concrete for Structural Analysis

After a basic structural system is configured, member sizes selected, and loads determined, the structure is analyzed to obtain internal force demands. For simple structures, analysis by hand calculations or approximate methods would suffice (see Section 7.8); otherwise, structural analysis software may be used. For most reinforced concrete structures, a linear elastic analysis, assuming the gross moment of inertia of cross-sections and neglecting the steel reinforcement area, will provide results of sufficient accuracy for design purposes. The final design will generally be conservative even though the analysis does not reflect the actual nonlinear structural behavior because member design is based on ultimate strength design and the ductility of reinforced concrete enables force redistributions (see Sections 7.9 and 7.11). Refined modeling using nonlinear analysis is generally not necessary unless it is a special type of structure under severe loading situations like high seismic forces.

For structural modeling, the concrete modulus E_c given in Table 7.2 can be used for input. When the ends of beam and column members are cast together, the rigid end zone modeling option should be selected since its influence is often significant. Reinforced concrete floor systems should be modeled as rigid diaphragms by master slaving the nodes on a common floor. Tall walls or cores can be modeled as column elements. Squat walls should be modeled as plate or shear wall elements. If foundation conditions and soil conditions are exceptional, then the foundation system will need more refined modeling. Otherwise, the structural model can be assumed to be fixed to the ground. For large reinforced concrete systems or when geometrical control is important, the effects of creep and shrinkage and construction staging should be incorporated in the analysis.

If slender columns are present in the structure, a second-order analysis should be carried out that takes into account cracking by using reduced or effective cross-sectional properties (see Table 7.3 and Section 7.14). If a refined model and nonlinear analysis is called for, then the moment curvature analysis results will be needed for input into the computer analysis (see Section 7.10).

7.8 Approximate Analysis of Continuous Beams and One-Way Slabs

Under typical conditions, for continuous beams and one-way slabs with more than two spans the approximate moment and shear values given in Figure 7.2 may be used in lieu of more accurate analysis methods. These values are from ACI 8.3.3.

TABLE 7.3 Suggested Effective Member Properties for Analysis

Member	Effective moment of inertia for analysis
Beam	$0.35I_g$
Column	$0.70I_g$
Wall — uncracked	$0.70I_g$
Wall — cracked	$0.35I_g$
Flat plates and flat slabs	$0.25I_g$

Note: I_g is the gross uncracked moment of inertia. Use gross areas for input of cross-sectional areas.

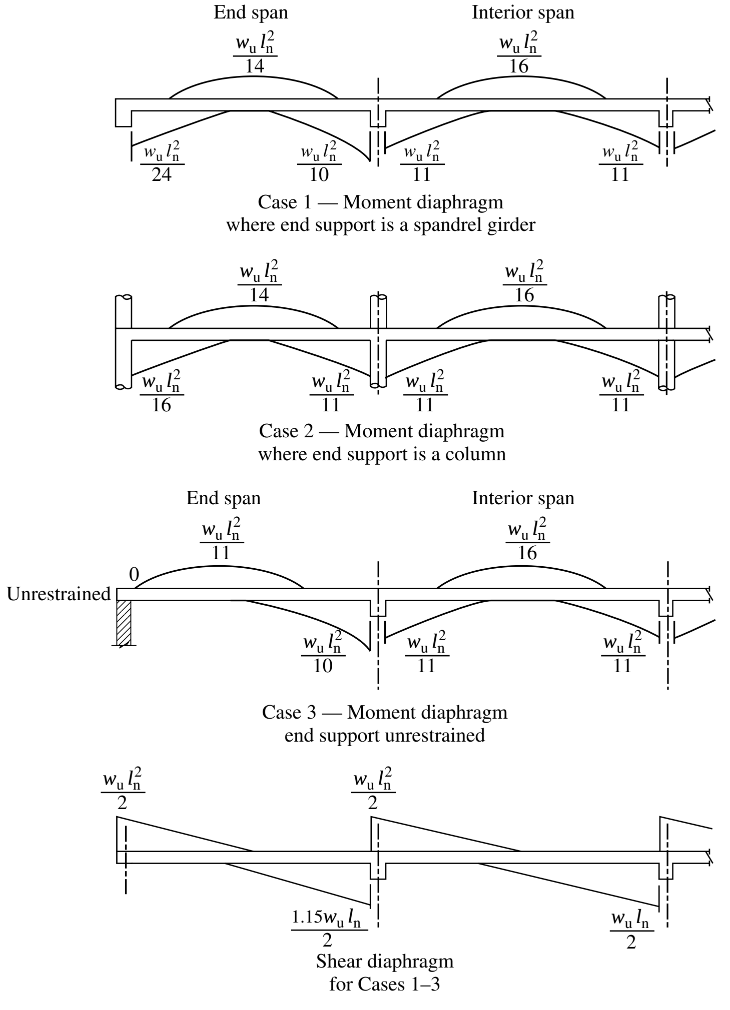

FIGURE 7.2 Approximate moment and shear of continuous beams or one-way slabs (ACI 8.3.3).

7.9 Moment Redistribution

The moment values of a continuous beam obtained from structural analysis may be adjusted or redistributed according to guidelines set by ACI 8.4. Negative moment can be adjusted down or up, but not more than $1000\varepsilon_t$ or 20% (see Notation section for ε_t). After the negative moments are adjusted in a span, the positive moment must also be adjusted to maintain the statical equilibrium of the span (see Section 7.13.12). Redistribution of moment is permitted to account for the ductile behavior of reinforcement concrete members.

7.10 Second-Order Analysis Guidelines

When a refined second-order analysis becomes necessary, as in the case where columns are slender, ACI 10.10.1 places a number of requirements on the analysis.

1. The analysis software should have been validated with test results of indeterminate structures and the predicted ultimate load within 15% of the test results.

2. The cross-section dimensions used in the analysis model must be within 10% of the dimensions shown in the design drawings.
3. The analysis should be based on factored loads.
4. The analysis must consider the material and geometrical nonlinearity of the structure, as well as the influence of cracking.
5. The effects of long-term effects, such as creep shrinkage and temperature effects, need to be assessed.
6. The effect of foundation settlement and soil–structure interaction needs to be evaluated.

A number of commercial software are available that meet the first requirement. If the second requirement is not met, the analysis must be carried out a second time. For the fourth requirement, the moment–curvature or moment–rotation curves need to be developed for the members to provide the accurate results. Alternatively, the code permits approximating the nonlinear effects by using the effective moment of inertias given in Table 7.3. Under the long-term influences of creep and shrinkage, and for stability checks, the effective moment of inertia needs to be further reduced by dividing it by $(1+\beta_d)$.

7.11 Moment–Curvature Relationship of Reinforced Concrete Members

Member curvature ϕ can be defined as rotation per unit length. It is related to the applied moment M and the section stiffness by the relationship $EI = M/\phi$. A typical moment–curvature diagram of a reinforced concrete beam is shown in Figure 7.3. The reduction in slope of the curve (EI) is the result of concrete cracking and steel yielding. The moment–curvature relationship is a basic parameter of deformation. This information is needed for input if a nonlinear analysis is carried out. For an unconfined reinforced concrete beam section, the point of first cracking is usually

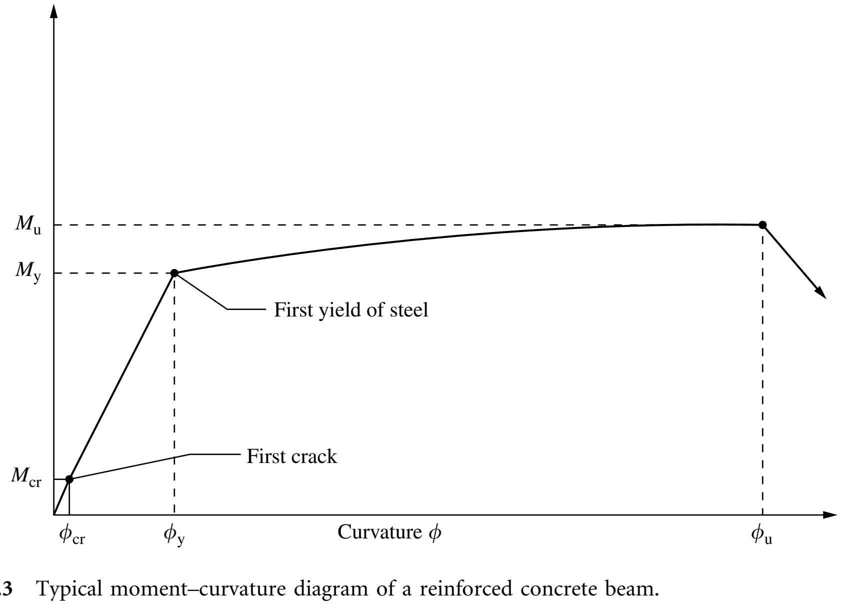

FIGURE 7.3 Typical moment–curvature diagram of a reinforced concrete beam.

neglected for input; the curvature points of first yield ϕ_y and ultimate ϕ_u are calculated from the following formulas:

$$\phi_y = \frac{f_y/E_s}{d(1-k)} \tag{7.1}$$

where

$$k = \left[(\rho+\rho')^2 n^2 + 2\left(\rho + \frac{\rho' d'}{d}\right)n\right]^{1/2} - (\rho+\rho')n \tag{7.2}$$

At ultimate

$$\phi_u = \frac{0.85\beta_1 E_s f_c'}{f_y^2(\rho-\rho')}\varepsilon_c\left\{1 + (\rho+\rho')n - \left[(\rho+\rho')^2 n^2 + 2\left(\rho + \frac{\rho' d'}{d}\right)n\right]^{1/2}\right\} \tag{7.3}$$

The concrete strain at ultimate ε_c is usually assumed to be a value between 0.003 and 0.004 for unconfined concrete. Software is available to obtain more refined moment–curvature relationships and to include other variables. If the concrete is considered confined, then an enhanced concrete stress–strain relationship may be adopted. For column members, the strain compatibility analysis must consider the axial load.

7.12 Member Design for Strength

7.12.1 Ultimate Strength Design

The main requirement of structural design is for the structural capacity, S_C, to be equal to or greater than the structural demand, S_D:

$$S_C \geq S_D$$

Modification factors are included in each side of the equation. The structural capacity S_C is equal to the nominal strength F_n multiplied by a capacity reduction safety factor ϕ:

$$S_C = \phi F_n$$

The nominal strength F_n is the internal ultimate strength at that section of the member. It is usually calculated by the designer according to formulas derived from the theory of mechanics and strength of materials. These strength formulas have been verified and calibrated with experimental testing. They are generally expressed as a function of the cross-section geometry and specified material strengths. There are four types of internal strengths: nominal moment M_n, shear V_n, axial P_n, and torsional moment T_n.

The capacity reduction safety factor ϕ accounts for uncertainties in the theoretical formulas, empirical data, and construction tolerances. The ϕ factor values specified by ACI are listed in Table 7.4.

The structural demand, S_D, is the internal force (moment, shear, axial, or torsion) at the section of the member resulting from the loads on the structure. The structural demand is usually obtained by carrying out a structural analysis of the structure using hand, approximate methods, or computer software. Loads to be input are specified by the design codes and the project specifications and normally include dead, live, wind, and earthquake loads. Design codes such and ACI, IBC, and AASHTO also specify the values of safety factors that should be multiplied with the specified loads and how different types of loads should be combined (i.e., $S_D = 1.2\text{Dead} + 1.6\text{Live}$). ACI load factors and combinations are listed in Table 7.5.

Combining the two equations above, a direct relationship between the nominal strength F_n and the structural demand S_D can be obtained

$$F_n \geq S_D/\phi \tag{7.4}$$

This relationship is convenient because the main design variables, such as reinforcement area, which are usually expressed in terms F_n, can be related directly to the results of the structural analysis.

TABLE 7.4 ACI Strength Reduction Factors ϕ

Nominal strength condition	Strength reduction factor ϕ
Flexure (tension-controlled)	0.90
Compression-controlled (columns)	
Spiral transverse reinforcement	0.70[a]
Other transverse reinforcement	0.65[a]
Shear and torsion	0.75
Bearing on concrete	0.65
Structural plain concrete	0.55

[a] ϕ is permitted to be linearly increased to 0.90 as the tensile strain in the extreme steel increases from the compression-controlled strain of 0.005.

Note: Under seismic conditions strength reduction factors may require modifications.

TABLE 7.5 ACI Load Factors

Load case	Structurals demand S_D or (required strength U)
1	$1.4(D+F)$
2	$1.2(D+F+T)+1.6(L+H)+0.5(L_r$ or S or $R)$
3	$1.2D+1.6(L_r$ or S or $R)+(1.0L$ or $0.8W)$
4	$1.2D+1.6W+1.0L+0.5(L_r$ or S or $R)$
5	$1.2D+1.0E+1.0L+0.2S$
6	$0.9D+1.6W+1.6H$
7	$0.9D+1.0E+1.6H$

Note: D is the dead load, or related internal moments and forces, E is the seismic load, F is the weight and pressure of well-defined fluids, H is the weight and pressure of soils, water in soil, or other materials, L is the live load, L_r is the roof live load, R is the rain load, S is the snow load, T is the time-dependent load (temperature, creep, shrinkage, differential settlement, etc.), and W is the wind load.

7.12.2 Beam Design

The main design steps for beam design and the formulas for determining beam capacity are outlined in the following.

7.12.2.1 Estimate Beam Size and Cover

Table 7.6 may be referenced for selecting a beam thickness. For practical construction, the minimum width of a beam is about 12 in. Economical designs are generally provided when the beam width to thickness ratio falls in the range of $\frac{1}{2}$ to 1. Minimum concrete covers are listed in Table 7.7 and typically should not be less than 1.5 in.

7.12.2.2 Moment Capacity

Taking a beam segment, flexural bending induces a force couple (see Figure 7.4). Internal tension N_T is carried by the reinforcement (the tensile strength of concrete is low and its tension carrying capacity is neglected). Reinforcement at the ultimate state is required to yield, hence

$$N_T = A_s f_y \tag{7.5}$$

At the opposite side of the beam, internal compression force N_C is carried by the concrete. Assuming a simplified rectangular stress block for concrete (uniform stress of $0.85f'_c$),

$$N_C = 0.85 f'_c ab \tag{7.6}$$

To satisfy equilibrium, internal tension must be equal to internal compression, $N_C = N_T$. Hence, the depth of the rectangular concrete stress block a can be expressed as

$$a = \frac{A_s f_y}{0.85 f'_c b} \tag{7.7}$$

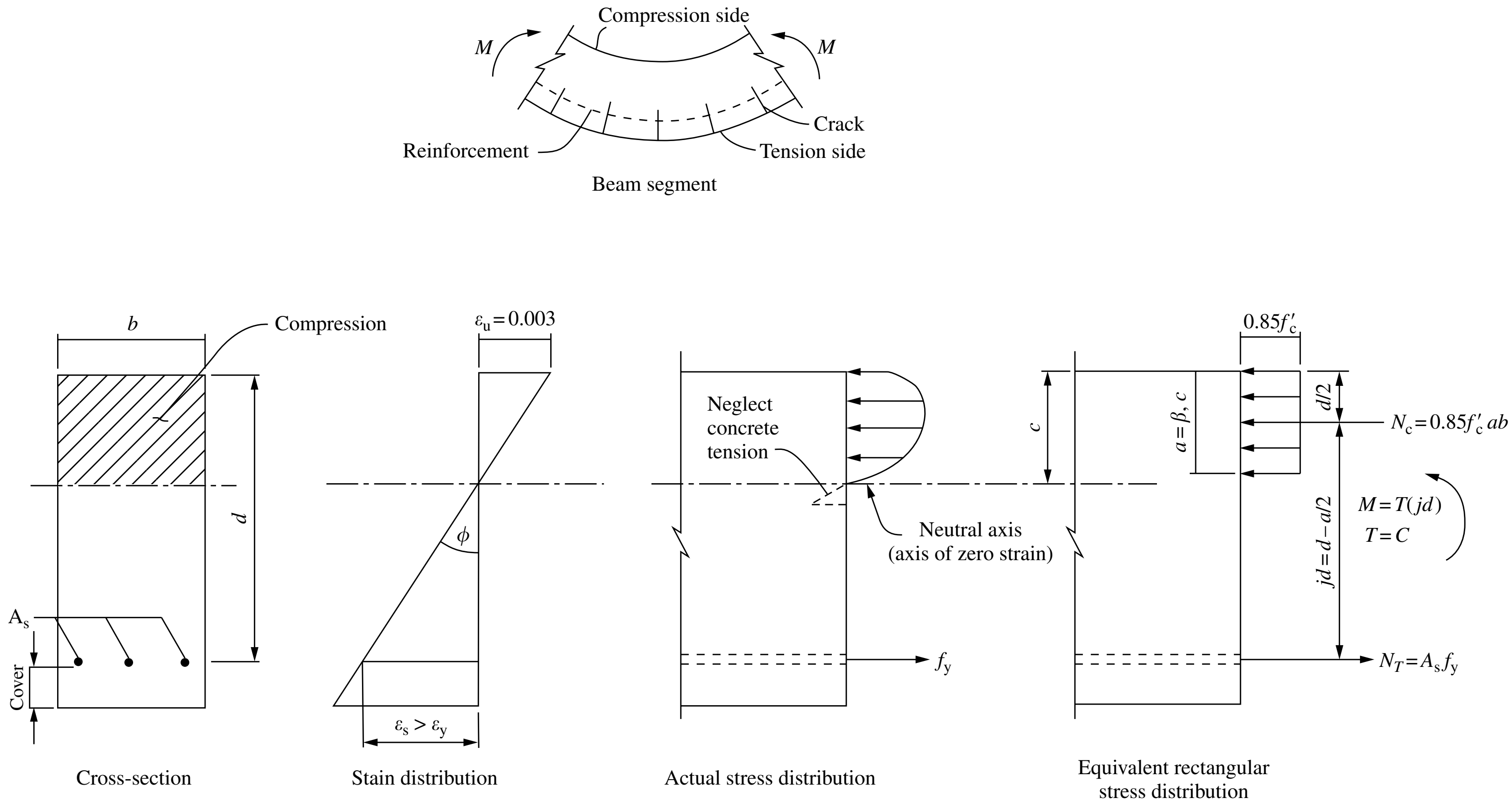

FIGURE 7.4 Mechanics of reinforced concrete beam under flexure.

TABLE 7.6 Minimum Depth of Beams

	Minimum thickness, h			
	Support condition (L = span length)			
Member	Simply supported	One end continuous	Both ends continuous	Cantilever
Beams or one-way joists	$L/16$	$L/18.5$	$L/21$	$L/8$
One-way slabs	$L/20$	$L/24$	$L/28$	$L/10$

Notes:

1. Applicable to normal-weight concrete members reinforced with Grade 60 steel and members not supported or attached to partitions or other construction likely to be damaged by large deflection.
2. For reinforcement f_y other than 60,000 psi, the h values above should be multiplied by $(0.4 + f_y/100{,}000)$.
3. For lightweight concrete of weight W_c (pcf), the h values above should be multiplied by $(1.65 - 0.005W_c)$, but should not be less than 1.09.

TABLE 7.7 Minimum Concrete Cover

Exposure condition and member type	Minimum cover (in.)
Concrete not exposed to weather or in contact with ground	
Beams, columns	$1\frac{1}{2}$
Slabs, joist, walls	
No. 11 bar and smaller	$\frac{3}{4}$
No. 14 and No. 18 bars	$1\frac{1}{2}$
Concrete exposed to weather or earth	
No. 5 bar and smaller	$1\frac{1}{2}$
No. 6 through No. 18 bars	2
Concrete cast against and permanently exposed to earth	3

The moment capacity of the beam section ϕM_n may be expressed as the tension force multiplied by the moment arm of the force couple.

$$\phi M_n = \phi A_s f_y \left(d - \frac{a}{2}\right) \tag{7.8}$$

The strength reduction factor for flexure ϕ is 0.9.

7.12.2.3 Determination of Required Flexural Reinforcement Area

The maximum moment demand is determined from the structural analysis of the structure under the specified loads and load combinations, M_u. The nominal moment capacity M_n that the cross-section must supply is therefore

$$M_n = M_u/\phi \tag{7.9}$$

The beam cross-section dimensions, width b and thickness h, would be determined first or a first trial selected; the depth of the beam to the centroid of the tension reinforcement can be estimated by

$$d = h - \text{concrete cover} - \text{stirrup diameter} - \text{tension reinforcement bar radius} \tag{7.10}$$

A reasonable size of the stirrup and reinforcement bar can be assumed, if not known (a No. 4 or No. 5 bar size for stirrups is reasonable).

Rearranging the moment capacity equations presented in the previous section, the required flexural reinforcement is obtained by solving for A_s

$$A_s = \frac{M_n}{f_y\left(d - \frac{1}{2}\left(A_s f_y/0.85 f_c' b\right)\right)} \tag{7.11}$$

The required tension reinforcement area A_s is obtained from the quadratic expression

$$A_s = \frac{f_y d \pm \sqrt{\left(f_y d\right)^2 - 4 M_n K_m}}{2 K_m} \tag{7.12}$$

where K_m is a material constant:

$$K_m = \frac{f_y^2}{1.7 f_c' b} \tag{7.13}$$

Then, the sizes and quantity of bars are selected. Minimum requirements for reinforcement area and spacing must be satisfied (see the next two sections).

7.12.2.4 Limits on Flexural Reinforcement Area

1. *Minimum reinforcement area for beams:*

$$A_{s,\min} = \frac{3\sqrt{f_c'}}{f_y} b_w d \geq 200 b_w d/f_y \tag{7.14}$$

2. *Maximum reinforcement for beams:* The maximum reinforcement A_s must satisfy the requirement that the net tensile strain ε_t (extreme fiber strain less effects of creep, shrinkage, and temperature) is not less than 0.004. The net tensile strain is solved from the compatibility of strain (see Figure 7.4).

$$\varepsilon_t = 0.003 \frac{d - c}{c} \tag{7.15}$$

The neutral axis location c is related to the depth of the compression stress block a by the relationship (ACI 10.2.7.3)

$$c = a/\beta_1 \tag{7.16}$$

The factor β_1 is dependent on the concrete strength as shown in Figure 7.5.

7.12.2.5 Detailing of Longitudinal Reinforcement

Clear spacing between parallel bars should be large enough to permit the coarse aggregate to pass through to avoid honeycombing. The minimum clear spacing should be d_b, but it should not be less than 1 in.

For crack control, center-to-center spacing of bars should not exceed

$$\frac{540}{f_s} - 2.5 c_c \leq \frac{432}{f_s} \tag{7.17}$$

where f_s (in ksi) is the stress in the reinforcement at service load, which may be assumed to be 60% of the specified yields strength. Typically, the maximum spacing between bars is about 10 in. The maximum bar spacing rule ensures that crack widths fall below approximately 0.016 in. For very aggressive exposure environments, additional measures should be considered to guard against corrosion, such as reduced concrete permeability, increased cover, or application of sealants.

If the depth of the beam is large, greater than 36 in., additional reinforcement should be placed at the side faces of the tension zone to control cracking. The amount of skin reinforcement to add need not exceed one half of the flexural tensile reinforcement and it should be spread out for a distance $d/2$. The spacing of the skin reinforcement need not exceed $d/6$, 12 in., and $1000A_b/(d - 30)$.

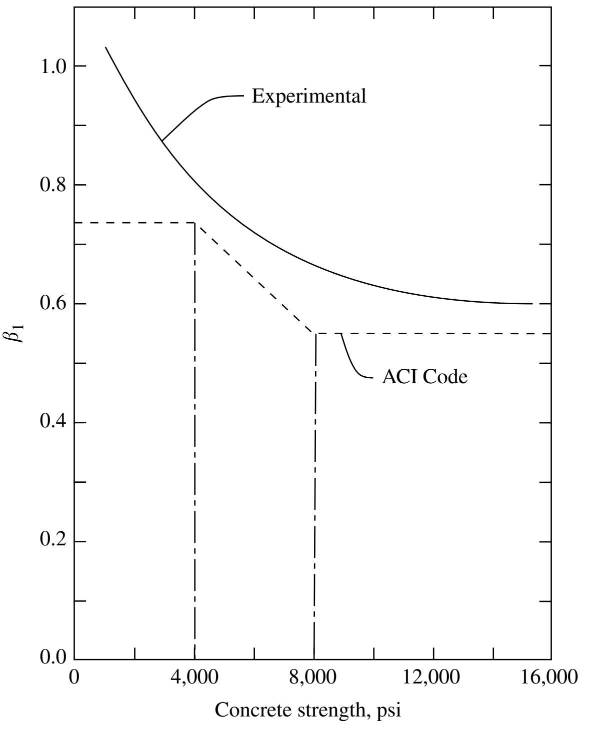

FIGURE 7.5 Relation between β_1 and concrete strength.

To ease reinforcement cage fabrication, a minimum of two top and two bottom bars should run continuously through the span of the beam. These bars hold up the transverse reinforcement (stirrups). At least one fourth of all bottom (positive) reinforcement should run continuously. If moment reversal is expected at the beam–column connection, that is, stress reversal from compression to tension, bottom bars must be adequately anchored into the column support to develop the yield strength.

The remaining top and bottom bars may be cut short. However, it is generally undesirable to cut bars within the tension zone (it causes loss of shear strength and ductility). It is good practice to run bars well into the compression zone, at least a distance d, $12d_b$ or $l_n/16$ beyond the point of inflection (PI) (see Figure 7.6). Cut bars must also be at least one development length l_d in length measured from each side of their critical sections, which are typically the point of peak moment where the yield strength must be developed. See Section 7.17 for development lengths.

To achieve structural integrity of the structural system, beams located at the perimeter of the structure should have minimum continuous reinforcement that ties the structure together to enhance stability, redundancy, and ductile behavior. Around the perimeter at least one sixth of the top (negative) longitudinal reinforcement at the support and one quarter of the bottom (positive) reinforcement should be made continuous and tied with closed stirrups (or open stirrups with minimum 135° hooks). Class A splices may be used to achieve continuity. Top bars should be spliced at the midspan, bottom bars at or near the support.

7.12.2.6 Beams with Compression Reinforcement

Reinforcement on the compression side of the cross-section (see Figure 7.4) usually does not increase in flexural capacity significantly, typically less than 5%, and for most design purposes its contribution to

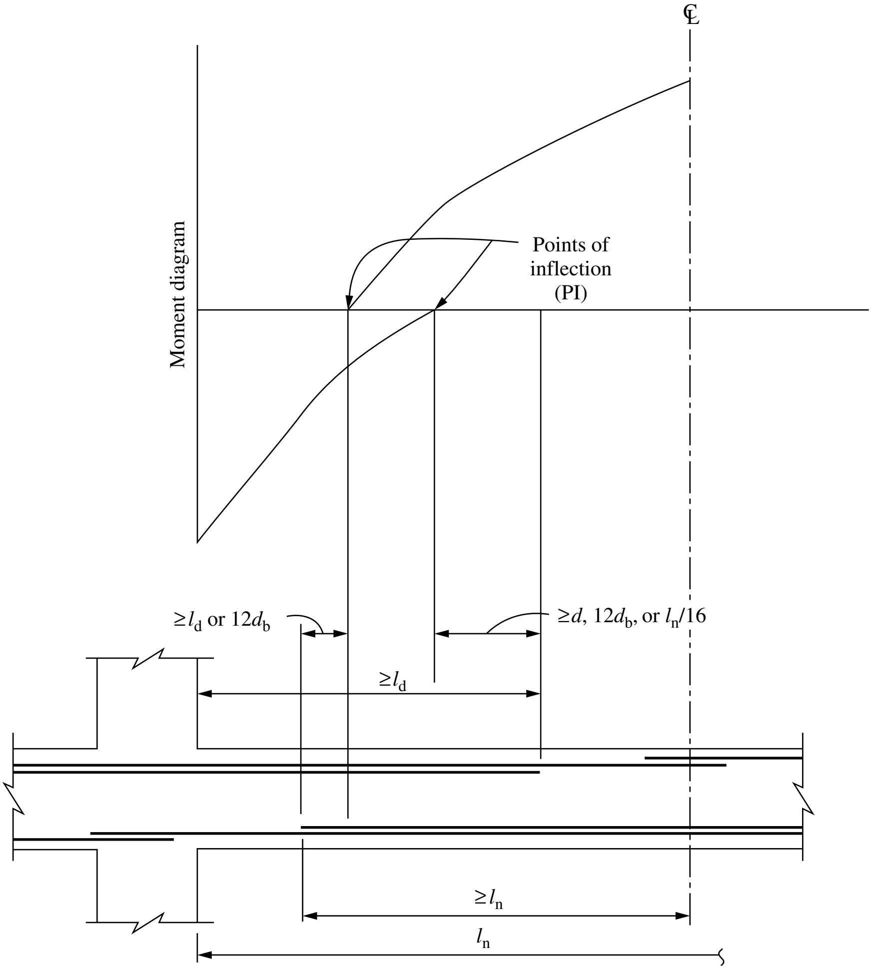

FIGURE 7.6 Typical reinforcement cutoffs for continuous beam.

strength can be neglected. The moment capacity equation considering the compression reinforcement area A_s' located at a distance d' from the compression fiber is

$$\phi M_n = A_s' f_y (d - d') + (A_s - A_s') f_y \left(d - \frac{a}{2}\right) \tag{7.18}$$

where

$$a = \frac{[A_s - A_s'(1 - 0.85 f_c'/f_y)] f_y}{0.85 f_c' b} \tag{7.19}$$

The above expressions assume the compression steel yield, which is typically the case (compression steel quantity is not high). For the nonyielding case, the stress in the steel needs to be determined by a stress–strain compatibility analysis.

Despite its small influence on strength, compression reinforcement serves a number of useful serviceability functions. It is needed for supporting the transverse shear reinforcement in the fabrication of the steel cage. It helps to reduce deflections and long-term creep, and it enhances ductile performance.

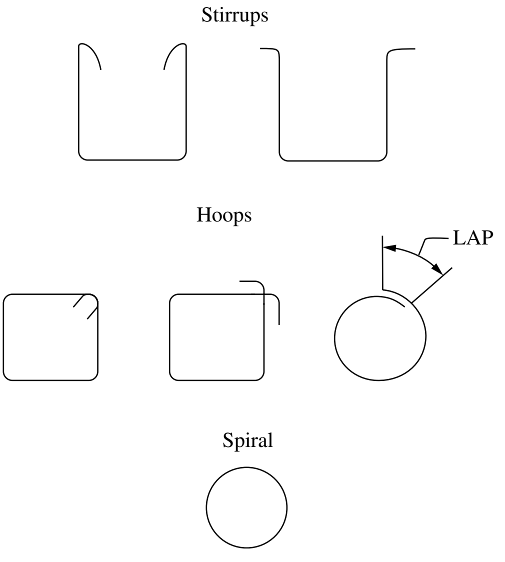

FIGURE 7.7 Typical types of transverse reinforcement.

7.12.2.7 Shear Capacity of Beams

Shear design generally follows after flexural design. The shear capacity ϕV_n of a beam consists of two parts: (1) the shear provided by the concrete itself V_c and (2) that provided by the transverse reinforcement V_s.

$$\phi V_n = \phi(V_c + V_s) \tag{7.20}$$

The strength reduction factor ϕ for shear is 0.85. The nominal shear capacity of the concrete may be taken as the simple expression

$$V_c = 2\sqrt{f_c'}\, b_w d \tag{7.21a}$$

which is in pound and inch units. An alternative empirical formula that allows a higher concrete shear capacity is

$$V_c = \left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_u}\right) b_w d \leq 3.5\sqrt{f_c'}\, b_w d \tag{7.21b}$$

where M_u is the factored moment occurring simultaneously with V_u at the beam section being checked. The quantity $V_u d/M_u$ should not be taken greater than 1.0.

Transverse shear reinforcements are generally of the following types (see Figure 7.7): stirrups, closed hoops, spirals, or circular ties. In addition, welded wire fabric, inclined stirrups, or longitudinal bars bent at an angle may be used. For shear reinforcement aligned perpendicular to the longitudinal reinforcement, the shear capacity provided by transverse reinforcement is

$$V_s = \frac{A_v f_y d}{s} \leq 8\sqrt{f_c'}\, b_w d \tag{7.22a}$$

When spirals or circular ties or hoops are used with this formula, d should be taken as 0.8 times the diameter of the concrete cross-section, and A_v should be taken as two times the bar area.

When transverse reinforcement is inclined at an angle α with respect to the longitudinal axis of the beam, the transverse reinforcement shear capacity becomes

$$V_s = \frac{A_v f_y (\sin\alpha_i + \cos\alpha_i) d}{s} \leq 8\sqrt{f_c'}\, b_w d \tag{7.22b}$$

The shear formulas presented above were derived empirically, and their validity has also been tested by many years of design practice. A more rational design approach for shear is the strut-and-tie model, which is given as an alternative design method in ACI Appendix A. Shear designs following the strut-and-tie approach, however, often result in designs requiring more transverse reinforcement steel since the shear transfer ability of concrete is neglected.

7.12.2.8 Determination of Required Shear Reinforcement Quantities

The shear capacity must be greater than the shear demand V_u, which is based on the structural analysis results under the specified loads and governing load combination

$$\phi V_n \geq V_u \tag{7.23}$$

Since the beam cross-section dimensions b_w and d would usually have been selected by flexural design beforehand or governed by functional or architectural requirements, the shear capacity provided by the concrete V_c can be calculated by Equations 7.21a or 7.21b. From the above equations, the required shear capacity to be provided by shear reinforcement must satisfy the following:

$$V_s \geq \frac{V_u}{\phi} - V_c \tag{7.24}$$

Inserting V_s from this equation into Equation 7.22a, the required spacing and bar area of the shear reinforcement (aligned perpendicular to the longitudinal reinforcement) must satisfy the following:

$$\frac{s}{A_v} \leq \frac{f_y d}{V_s} \tag{7.25}$$

For ease of fabrication and bending, a bar size in the range of No. 4 to No. 6 is selected, then the required spacing s along the length of the beam is determined, usually rounded down to the nearest $\frac{1}{2}$ in.

In theory, the above shear design procedure can be carried out at every section along the beam. In practice, a conservative approach is taken and shear design is carried out at only one or two locations of maximum shear, typically at the ends of the beam, and the same reinforcement spacing s is adopted for the rest of the beam. Where the beam ends are cast integrally or supported by a column, beam, wall, or support element that introduces a region of concentrated compression, the maximum value of the shear demand need not be taken at the face of the support, but at a distance d away (see Figure 7.8).

Transverse reinforcement in the form of closed stirrups is preferred for better ductile performance and structural integrity. For beams located at the perimeter of the structure, ACI requires closed stirrups (or open stirrups within minimum 135° hooks). In interior beams, if closed stirrups are not provided, at least one quarter of the bottom (positive) longitudinal reinforcement at midspan should be made continuous over the support, or at the end support, detailed with a standard hook.

7.12.2.8.1 Minimum Shear Reinforcement and Spacing Limits

After the shear reinforcement and spacing are selected they should be checked against minimum requirements. The minimum shear reinforcement required is

$$A_{vmin} = 0.75\sqrt{f_c'}\frac{b_w s}{f} \geq \frac{50 b_w s}{f_y} \tag{7.26}$$

This minimum shear area applies in the beam where $V_u \geq \phi V_2/2$. It does not apply to slabs, footings, and concrete joists. The transverse reinforcement spacing s should not exceed $d/2$ nor 24 in. These spacing limits become $d/4$ and 12 in. when V_s exceeds $4\sqrt{f_c'}b_w d$.

When significant torsion exists, additional shear reinforcement may be needed to resist torsion. This is covered in Section 7.16.

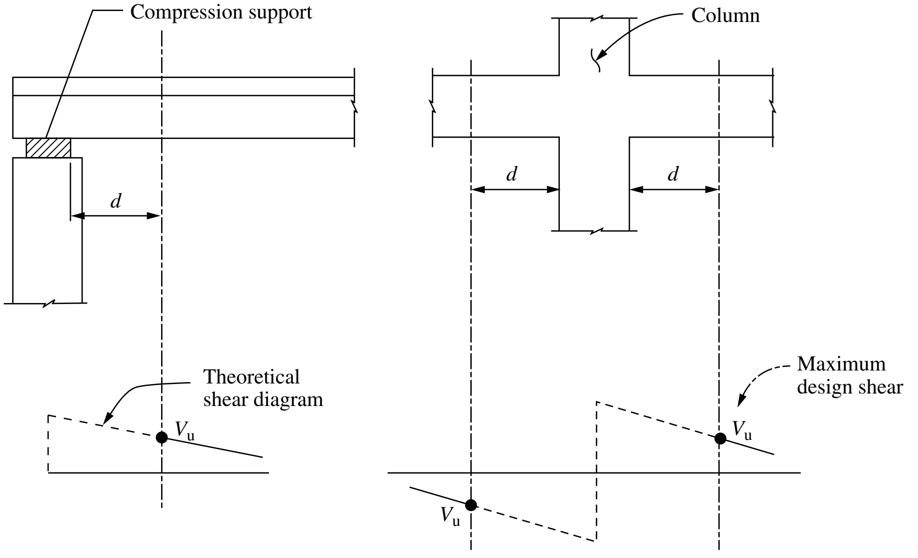

FIGURE 7.8 Typical support conditions for locating factored shear force V_u.

7.12.2.8.2 Modifications for High-Strength and Lightweight Concretes

For concretes with compressive strengths greater than 10,000 psi, the values of $\sqrt{f'_c}$ in all the shear capacity and design equations above should not exceed 100 psi. For lightweight concretes, $\sqrt{f'_c}$ should be multiplied by 0.75 for all-lightweight concrete, or 0.85 for sand-lightweight concrete. If the tensile strength f_{ct} of the concrete is specified, $\sqrt{f'_c}$ may be substituted by $f_{ct}/6.7$, but should not be greater than $\sqrt{f'_c}$.

7.12.2.9 Detailing of Transverse Reinforcement

Transverse reinforcement should extend close to the compression face of a member, as far as cover allows, because at ultimate state deep cracks may cause loss of anchorage. Stirrup should be hooked around a longitudinal bar by a standard stirrup hoop (see Figure 7.9). It is preferable to use transverse reinforcement size No. 5 or smaller. It is more difficult to bend a No. 6 or larger bar tightly around a longitudinal bar. For transverse reinforcement sizes No. 6, No. 7, and No. 8, a standard stirrup hook must be accompanied by a minimum embedment length of $0.014 d_b f_y/\sqrt{f'_c}$ measured between the midheight of the member and the outside end of the hook.

7.12.3 One-Way Slab Design

When the load normal to the surface of a slab is transferred to the supports primarily in one major direction, the slab is referred to as a one-way slab. For a slab panel supported on all four edges, one-way action occurs when the aspect ratio, the ratio of its long-to-short span length, is greater than 2. Under one-way action, the moment diagram remains essentially constant across the width of the slab. Hence, the design procedure of a one-way slab can be approached by visualizing the slab as an assembly of the same beam strip of unit width. This beam strip can be designed using the same design steps and formulas presented in the previous section for regular rectangular beams.

The required cover for one-way slab is less than for beams, typically $\frac{3}{4}$ in. The internal forces in one-way slabs are usually lower, so smaller bar sizes are used. The design may be controlled by the minimum temperature and shrinkage reinforcement. Shear is rarely a controlling factor for one-way slab design. Transverse reinforcement is difficult to install in one-way slabs. It is more economical to thicken or haunch the slab.

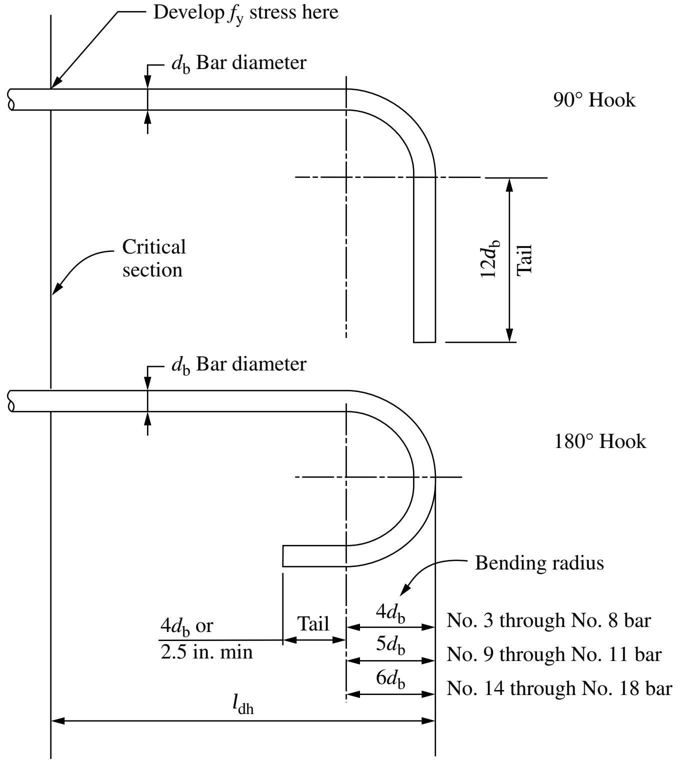

FIGURE 7.9 Standard hooked bar details.

7.12.3.1 Shrinkage and Temperature Reinforcement (ACI 7.12)

For Grade 60 reinforcement, the area of shrinkage and temperature reinforcement should be 0.0018 times the gross concrete area of the slab. Bars should not be spaced farther than five times the slab thickness or 18 in. The shrinkage and temperature requirements apply in both directions of the slab, and the reinforcement must be detailed with adequate development length where yielding is expected.

7.12.4 T-Beam Design

Where a slab is cast integrally with a beam, the combined cross-section acts compositely (see Figure 7.10). The design of T-beam differs from that of a rectangular beam only in the positive moment region, where part of the internal compression force occurs in the slab portion. The design procedures and formulas for T-beam design are the same as for rectangular beams, except for the substitution of b in the equations with an effective width b_{eff} at positive moment sections. The determination of b_{eff} is given in Figure 7.10. The effective width b_{eff} takes into account the participation of the slab in resisting compression. In the rare case where the depth of the compression stress block a exceeds the slab thickness, a general stress–strain compatibility analysis would be required. For shear design the cross-section width should be taken as the width of the web b_w.

7.12.4.1 Requirements for T-Beam Flanges

If the T-beam is an isolated beam and the flanges are used to provide additional compression area, the flange thickness should be not less than one half the width of the web and the effective flange width not more than four times the width of the web. For a slab that forms part of the T-beam flange and if the slab primary flexural reinforcement runs parallel to the T-beam, adequate transverse reinforcement needs

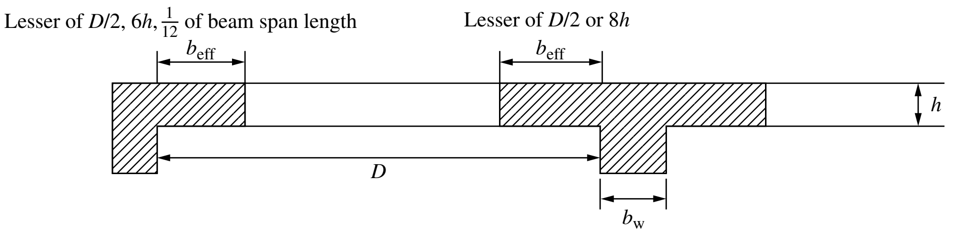

FIGURE 7.10 T-beam section.

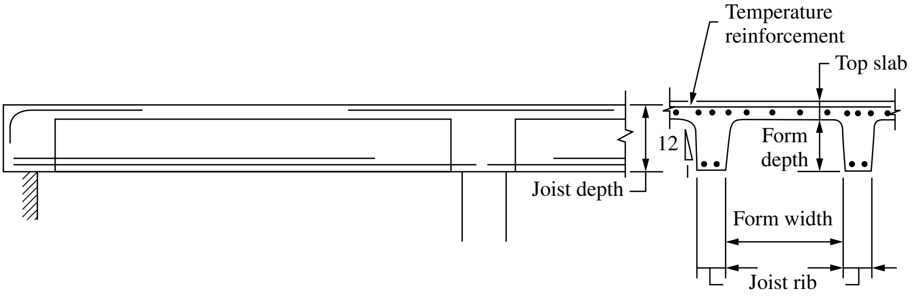

FIGURE 7.11 One-way joist.

to be provided in the slab by treating the flange as a cantilever. The full cantilevering length is taken for an isolated T-beam; otherwise, the effective flange length is taken.

7.12.5 One-Way Joist Design

A one-way joist floor system consists of a series of closely spaced T-beams (see Figure 7.11). The ribs of joists should not be less than 4 in. in width and should have a depth not more than 3.5 times the minimum width of the rib. Flexural reinforcement is determined by T-section design. The concrete ribs normally have sufficient shear capacity so that shear reinforcement is not necessary. A 10% increase is allowed in the concrete shear capacity calculation, V_c, if the clear spacing of the ribs does not exceed 30 in. Alternatively, higher shear capacity can be obtained by thickening the rib at the ends of the joist where the high shear demand occurs. If shear reinforcements are added, they are normally in the form of single-leg stirrups. The concrete forms or fillers that form the joists may be left in place; their vertical stems can be considered part of the permanent joist design if their compressive strength is at least equal to the joist. The slab thickness over the permanent forms should not be less than $\frac{1}{12}$ of the clear distance between ribs or less than 1.5 in. Minimum shrinkage and temperature reinforcement need to be provided in the slab over the joist stems. For structural integrity, at least one bottom bar in the joist should be continuous or spliced with a Class A tension splice (see Section 7.17) over continuous supports. At discontinuous end supports, bars should be terminated with a standard hook.

7.13 Two-Way Floor Systems

Design assuming one-way action is not applicable in many cases, such as when a floor panel is bounded by beams with a long to short aspect ratio of less than 2. Loads on the floor are distributed in both directions, and such a system is referred to as a two-way system (see Figure 7.12). The design approach of two-way floor systems remains in many ways similar to that of the one-way slab, except that the floor slab should now be visualized as being divided into a series of slab strips spanning *both* directions of the floor panel (see Figure 7.12). In the case of one-way slabs, each slab strip carries the same design moment diagram. In two-way systems, the design moment diaphragm varies from one strip to another. Slab strips

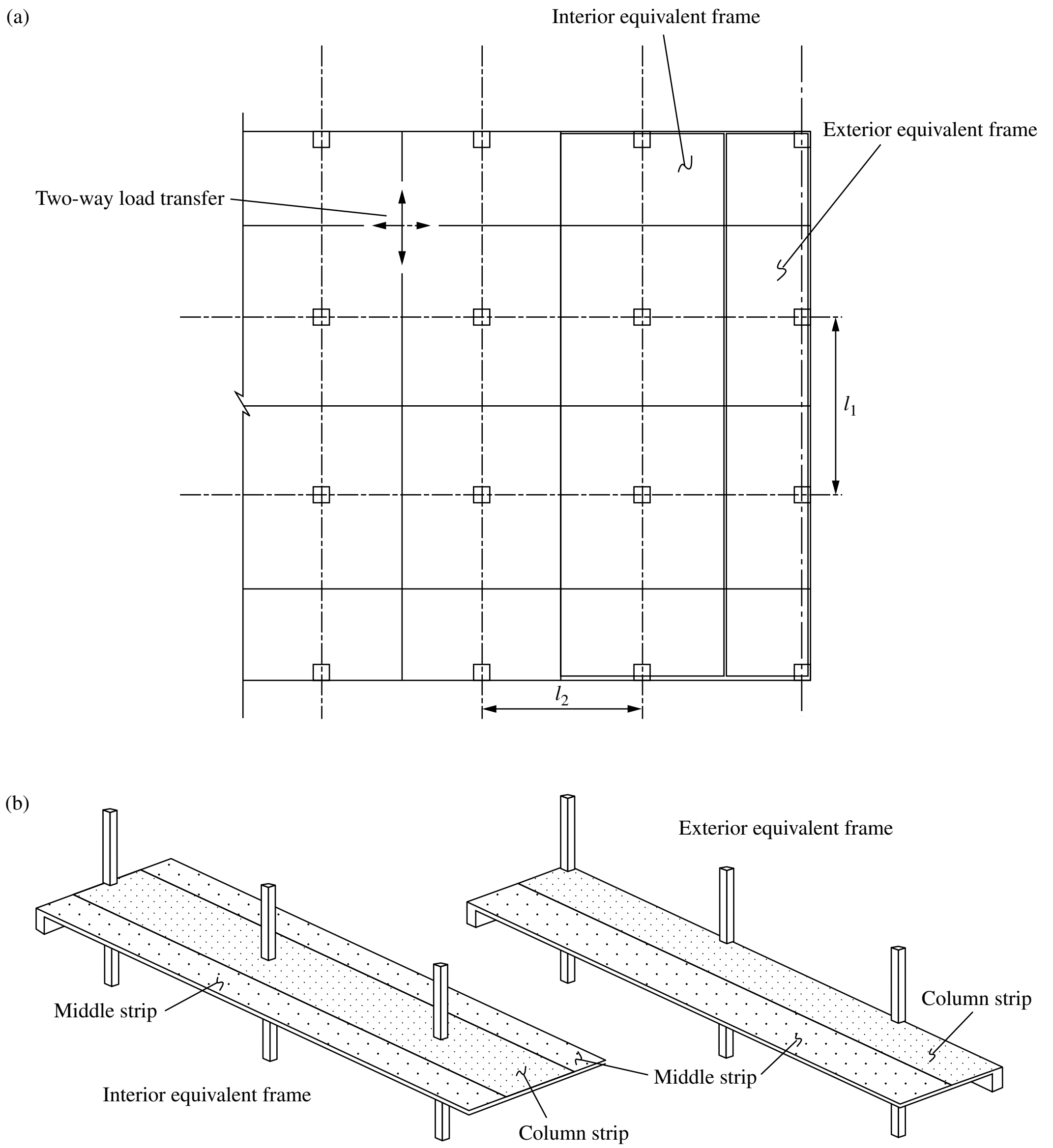

FIGURE 7.12 (a) Two-way floor system and (b) equivalent frames.

closer to the column support lines would generally carry a higher moment than strips at midspan. Hence, a key design issue for two-way floor design becomes one of analysis, on how to obtain an accurate estimate of internal force distribution among the slab strips. After this issue is resolved, and the moment diagrams of each strip are obtained, the flexural reinforcement design of each slab strip follows the same procedures and formulas as previously presented for one-way slabs and beams. Of course, the analysis of two-way floor systems can also be solved by computer software, using the finite element method, and a number of structural analysis software have customized floor slab analysis modules. The ACI Code contains an approximate manual analysis method, the Direct Design Method, for two-way floors, which is practical for design purposes. A more refined approximate method, the Equivalent Frame Method, is also available in the ACI.

If a floor system is regular in layout and stiffness (ACI 13.6.1), the Direct Design Method may be used to obtain the moment diagrams for the slab strips of two-way floor systems. The Direct Design Method is based on satisfying the global statical equilibrium of each floor panel. The relative stiffnesses of the panel components (e.g., slab, beam, drop panels) are then considered in distributing the statical moment. The subsequent sections present the application of the ACI Direct Design Method for different types of two-way systems.

General detailing of two-way slabs. The required slab reinforcement areas are taken at the critical sections, generally at the face of supports around the perimeter of the panel and at the midspans of the column and middle strips. The maximum spacing of reinforcement should not exceed two times the slab thickness or that required for temperature reinforcement (see Section 7.12.3.1). All bottom bars in slab panels that run perpendicular to the edge of the floor should be extended to the edge and anchored into the edge beam, column, or wall that exists there.

Opening in slabs of any size is permitted in the area common to intersecting middle strips (see Figure 7.12). But the original total reinforcement in the slab panel should be maintained by transferring bars to the sides of the opening. In intersecting column strips, not more than one eighth the width of the column strip should be interrupted by an opening. In the area common to one column strip and one middle strip, not more than one quarter of the reinforcement should be interrupted by an opening. If a larger opening is required, then edge beams or bands of reinforcement around the opening should be added.

7.13.1 Two-Way Slab with Beams

This system is shown in Figure 7.13. It consists of a slab panel bounded with beams supported on columns. Since the long to short aspect ratio of the panels is less than 2, a significant portion of the floor loading is transferred in the long direction. And the stiffness of the integral beams draws in load.

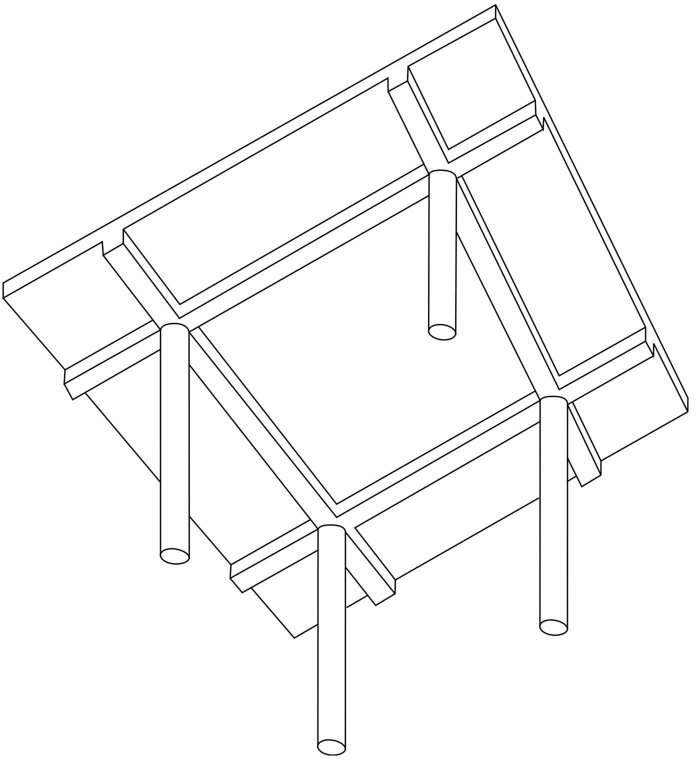

FIGURE 7.13 Two-way slab with beams.

TABLE 7.8 Minimum Thickness of Flat Plates (Two-Way Slabs without Interior Beams)

	Without drop panels			With drop panels		
	Exterior panels			Exterior panels		
Yield strength, f_y (psi)	Without edge beams	With edge beams	Interior panels	Without edge beams	With edge beams	Interior panels
40,000	$l_n/33$	$l_n/36$	$l_n/36$	$l_n/36$	$l_n/40$	$l_n/40$
60,000	$l_n/30$	$l_n/33$	$l_n/33$	$l_n/33$	$l_n/36$	$l_n/36$
75,000	$l_n/28$	$l_n/31$	$l_n/31$	$l_n/31$	$l_n/34$	$l_n/34$

Notes:
1. l_n is length of clear in long direction, face-to-face of support.
2. Minimum thickness if slabs without drop panels should not be less than 5 in.
3. Minimum thickness of slabs with drop panel should not be less than 4 in.

The minimum thickness of two-way slabs is dependent on the relative stiffness of the beams α_m. If $0.2 \leq \alpha_m \leq 2.0$, the slab thickness should not be less than 5 in. or

$$\frac{l_n\left(0.8 + (f_y/200{,}000\right)}{35 + 5\beta(\alpha_m - 0.2)} \tag{7.27}$$

If $\alpha_m > 2.0$, the denominator in the above equation should be replaced with $(36 + 9\beta)$, but the thickness should not be less than 3.5 in. When $\alpha_m < 0.2$, the minimum thickness is given by Table 7.8.

7.13.1.1 Column Strips, Middle Strips, and Equivalent Frames

For the Direct Design Method, to take into account the change of the moment across the panel, the floor system is divided into column and middle strips in each direction. The column strip has a width on each side of a column centerline equal to $0.25l_2$ or $0.25l_1$, whichever is less (see Figure 7.12). A middle strip is bounded by two column strips. The moment diagram across each strip is assumed to be constant and the reinforcement is designed for each strip accordingly.

In the next step of the Direct Design Method, equivalent frames are set up. Each equivalent frame consists of the columns and beams that share a common column or grid line. Beams are attached to the slabs that extend to the half-panel division on each side of the grid line, so the width of each equivalent frame consists of one column strip and two half middle strips (see Figure 7.12). Equivalent frames are set up for all the grid lines in both directions of the floor system.

7.13.1.2 Total Factored Static Moment

The first analysis step of the Direct Design Method is determining the total static moment in each span of the equivalent frame

$$M_0 = \frac{w_u l_2 l_n^2}{8} \tag{7.28}$$

Note that w_u is the full, not half, factored floor load per unit area. The clear span l_n is measured from face of column to face of column. The static moment is the absolute sum of the positive midspan moment plus the average negative moment in each span (see Figure 7.14).

The next steps of the Direct Design Method involve procedures for distributing the static moment M_0 into the positive (midspan) and negative moment (end span) regions, and then on to the column and middle strips. The distribution procedures are approximate and reflect the relative stiffnesses of the frame components (Table 7.9).

7.13.1.3 Distribution of Static Moment to Positive and Negative Moment Regions

The assignment of the total factored static moment M_0 to the negative and positive moment regions is given in Figure 7.14. For interior spans, $0.65M_0$ is assigned to each negative moment region and $0.35M_0$

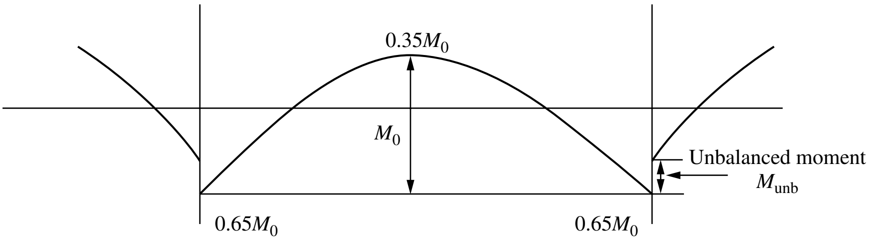

FIGURE 7.14 Static moment in floor panel.

TABLE 7.9 Distribution of Statical Moment for End Span Slab Panels

	Exterior edge unrestrained	Slab with beams between all supports	Slab without beams between interior supports: Without edge beam	Slab without beams between interior supports: With edge beam	Exterior edge fully restrained
Interior negative factored moment	0.75	0.70	0.70	0.70	0.65
Positive factored moment	0.63	0.57	0.52	0.50	0.35
Exterior negative factored moment	0	0.16	0.26	0.30	0.65

to the positive moment region. For the exterior span, the percentage of distribution is a function of the degree of restraint, as given in Table 7.9.

After the static moment is proportioned to the negative and positive regions, it is further apportioned on to the column and middle strips. For positive moment regions, the proportion of moment assigned to the column strips is given in Table 7.10. The parameter α_1 is a relative stiffness of the beam to slab, based on the full width of the equivalent frame:

$$\alpha_1 = \frac{E_{cb} I_b}{E_{cs} I_s} \tag{7.29}$$

For interior negative moment regions the proportion of moment assigned to the column strip is given by Table 7.11.

For negative moment regions of an exterior span, the moment assigned to the column follows Table 7.12, which takes into account the torsional stiffness of the edge beam. The parameter β_t is the ratio of torsional stiffness of edge beam section to flexural stiffness of a width of the slab equal to the center-to-center span length of the beam

$$\beta_t = \frac{E_{cb} C}{2 E_{cs} I_s} \tag{7.30}$$

The remaining moment, that was not proportioned to the column strips, is assigned to the middle strips.

Column strip moments need to be further divided into their slab and beam. The beam should be proportioned to take 85% of the column strip moment if $\alpha_I l_1 / l_2 \geq 1.0$. Linear interpolation is applied if this parameter is less than 1.0. If the beams are also part of a lateral force resisting system, then moments due to lateral forces should be added to the beams. After the assignment of moments, flexural reinforcement in the beams and slab strips can be determined following the same design procedures presented in Sections 7.12.2 and 7.12.3 for regular beams and one-way slabs.

TABLE 7.10 Distribution of Positive Moment in Column Strip

l_2/l_1	0.5	1.0	2.0
$(\alpha_1 l_2/l_1)=0$	60	60	60
$(\alpha_1 l_2/l_1)\geq 1.0$	90	75	45

TABLE 7.11 Distribution of Interior Negative Moment in Column Strip

l_2/l_1	0.5	1.0	2.0
$(\alpha_1 l_2/l_1)=0$	75	75	75
$(\alpha_1 l_2/l_1)\geq 1.0$	90	75	45

TABLE 7.12 Distribution of Negative Moment to Column of an Exterior Span

l_2/l_1		0.5	1.0	2.0
$(\alpha_1 l_2/l_1)=0$	$\beta_t=0$	100	100	100
	$\beta_t\geq 2.5$	75	75	75
$(\alpha_1 l_2/l_1)\geq 1.0$	$\beta_t=0$	100	100	100
	$\beta_t\geq 2.5$	90	75	45

7.13.1.4 Shear Design

The shear in the beam may be obtained by assuming that floor loads act according to the 45° tributary areas of each respective beam. Additional shear from lateral loads and the direct loads on the beam should be added on. The shear design of the beam then follows the procedure presented in Section 7.12. Shear stresses in the floor slab are generally low, but they should be checked. The strip method, which approximates the slab shear by assuming a unit width of slab strip over the panel, may be used to estimate the shear force in the slab.

7.13.2 Flat Plates

Floor systems without beams are commonly referred to as flat plates, (see Figure 7.15). Flat plates are economical and functional because beams are eliminated and floor height clearances are reduced. Minimum thicknesses of flat plates are given in Table 7.8 and should not be less than 5 in. The structural design procedure is the same as for flat slab with beams, presented in the previous sections, except that for flat plates $\alpha_1=0$. Refer to Section 7.13.1.2 for the static moment calculation. For the exterior span the distribution of the static moment is given in Figure 7.14. Table 7.10 and Table 7.11 provide the application for moment assignments to column strips.

7.13.2.1 Transfer of Forces in slab–column connections

An important design requirement of the flat plate system is the transfer of forces between the slab and its supporting columns (see Figure 7.14 and Figure 7.16). This transfer mechanism is a complex one. The accepted design approach is to assume that a certain fraction of the unbalanced moment M_{unb} in the slab connection is transferred by direct bending into the column support. This γ_f fraction is estimated to be

$$\gamma_f = \frac{1}{1+(2/3)\sqrt{b_1/b_2}} \tag{7.31}$$

The moment $\gamma_f M_{unb}$ is transferred over an effective slab width that extends 1.5 times the slab thickness outside each side face of the column or column capital support. The existing reinforcement in the column strip may be concentrated over this effective width or additional bars may be added.

The fraction of unbalanced moment not transferred by flexure γ_v ($\gamma_v = 1-\gamma_f$) is transferred through eccentricity of shear that acts over an imaginary critical section perimeter located at a

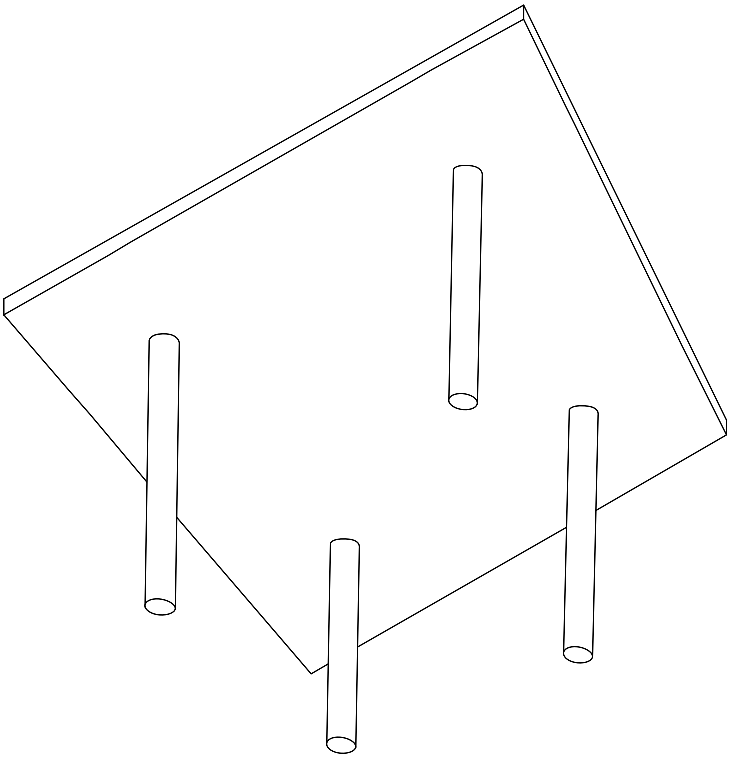

FIGURE 7.15 Flat plate.

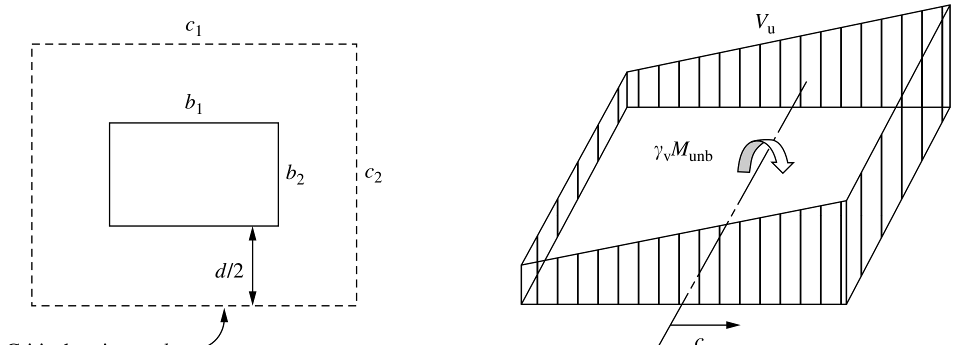

FIGURE 7.16 Transfer of shear in slab–column connections.

distance $d/2$ from the periphery of the column support (see Figure 7.16). Shear stress at the critical section is determined by combining the shear stress due to the direct shear demand V_u (which may be obtained from tributary loading) and that from the eccentricity of shear due to the unbalanced moment:

$$v_u = \frac{V_u}{A_c} \pm \frac{\gamma_v M_{unb} c}{J_c} \tag{7.32}$$

where the concrete area of the critical section $A_c = b_0 d = 2d(c_1 + c_2 + 2d)$, and J_c is the equivalent polar moment of inertia of the critical section

$$J_c = \frac{d(c_1 + d)^3}{6} + \frac{(c_1 + d)d^3}{6} + \frac{d(c_2 + d)(c_1 + d)^2}{2} \tag{7.33}$$

The maximum shear stress v_u on the critical section must not exceed the shear stress capacity defined by

$$\phi v_n = \phi V_c / b_0 d \tag{7.34}$$

The concrete shear capacity V_c for two-way action is taken to be the lowest of the following three quantities:

$$V_c = 4\sqrt{f_c'} b_0 d \tag{7.35}$$

$$V_c = \left(2 + \frac{4}{\beta_c}\right) 4\sqrt{f_c'} b_0 d \tag{7.36}$$

$$V_c = \left(\frac{\alpha_s d}{b_0} + 2\right) 4\sqrt{f_c'} b_0 d \tag{7.37}$$

where β_c is the ratio of long side to short side of the column. The factor α_s is 40 for interior columns, 30 for edge columns, or 20 for corner columns.

If the maximum shear stress demand exceeds the capacity, the designer should consider using a thicker slab or a larger column, or increasing the column support area with a column capital. Other options include insertion of shear reinforcement or shearhead steel brackets.

7.13.2.2 Detailing of Flat Plates

Refer to Figure 7.17 for minimum extensions for reinforcements. All bottom bars in the column strip should be continuous or spliced with a Class A splice. To prevent progressive collapse, at least two of the column strip bottom bars in each direction should pass within the column core or be anchored at the end supports. This provides catenary action to hold up the slab in the event of punching failure.

7.13.3 Flat Slabs with Drop Panels and/or Column Capitals

The capacity of flat plates may be increased with drop panels. Drop panels increase the slab thickness over the negative moment regions and enhance the force transfer in the slab–column connection. The minimum required configuration of drop panels is given in Figure 7.18. The minimum slab thickness is given in Table 7.8 and should not be less than 4 in.

Alternatively, or in combination with drop panels, column capitals may be provided to increase capacity (see Figure 7.19). The column capital geometry should follow a 45° projection. Column capitals increase the critical section of the slab–column force transfer and reduce the clear span lengths. The design procedure outlined for flat plates in the previous sections are applicable for flat slabs detailed with drop panels or column capitals.

7.13.4 Waffle Slabs

For very heavy floor loads or very long spans, waffle slab floor systems become viable (see Figure 7.20). A waffle slab can be visualized as being a very thick flat plate but with coffers to reduce weight and gain efficiency. The design procedure is therefore the same as for flat plates as presented in Section 7.13.2.

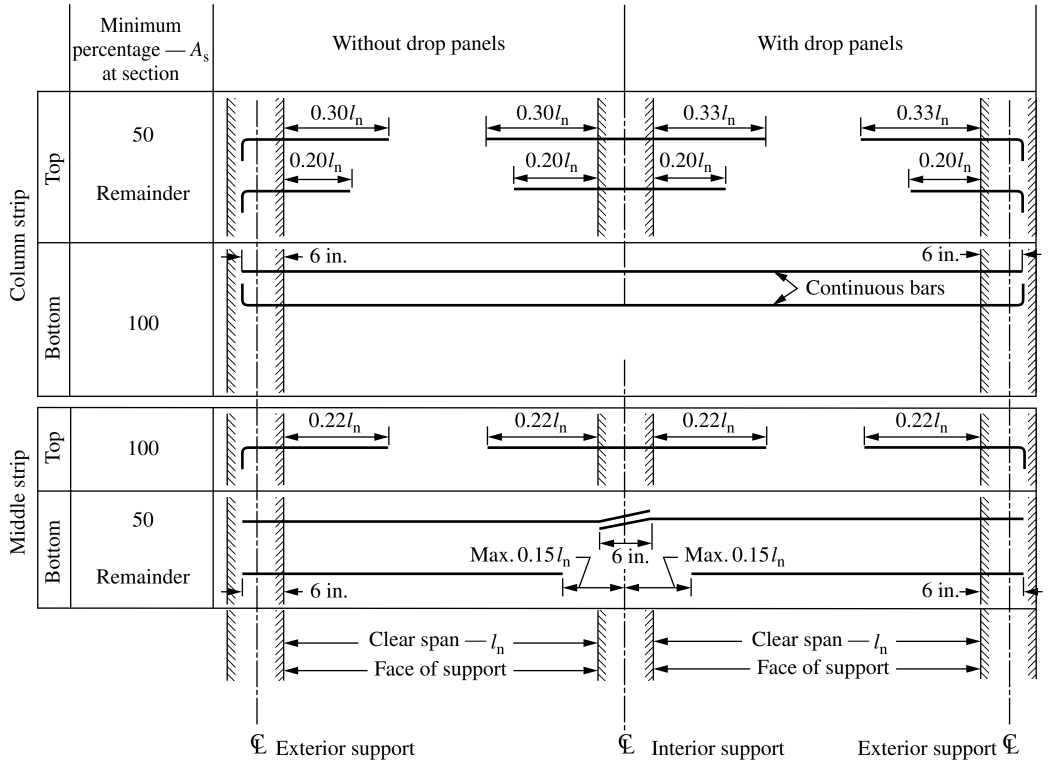

FIGURE 7.17 Detailing of flat plates.

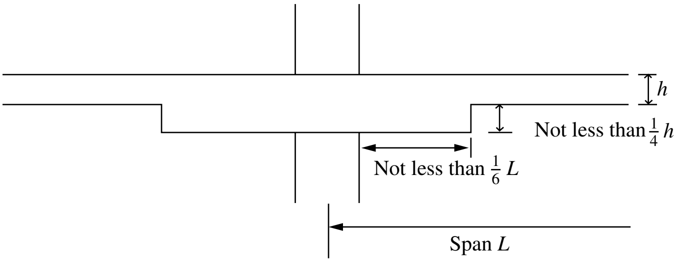

FIGURE 7.18 Drop panel dimensions.

The flexural reinforcement design is based on T-section strips instead of rectangular slab strips. Around column supports, the coffers may be filled in to act as column capitals.

7.14 Columns

Typical reinforcement concrete columns are shown in Figure 7.21. Longitudinal reinforcements in columns are generally distributed uniformly around the perimeter of the column section and run continuously through the height of the column. Transverse reinforcement may be in the form of rectangular hoops, ties, or spirals (Figure 7.21). Tall walls and core elements in buildings (Figure 7.22) are column-like in behavior and the design procedures presented in the following are applicable.

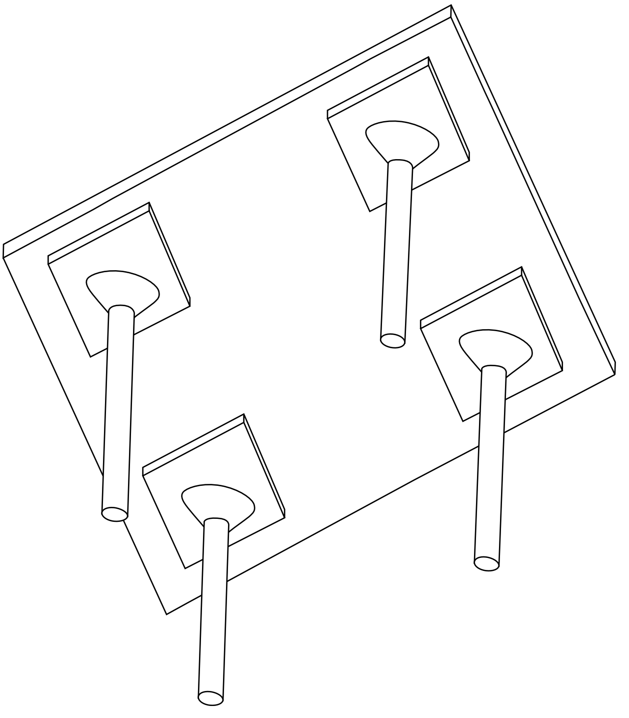

FIGURE 7.19 Flat slab with drop panels and column capital.

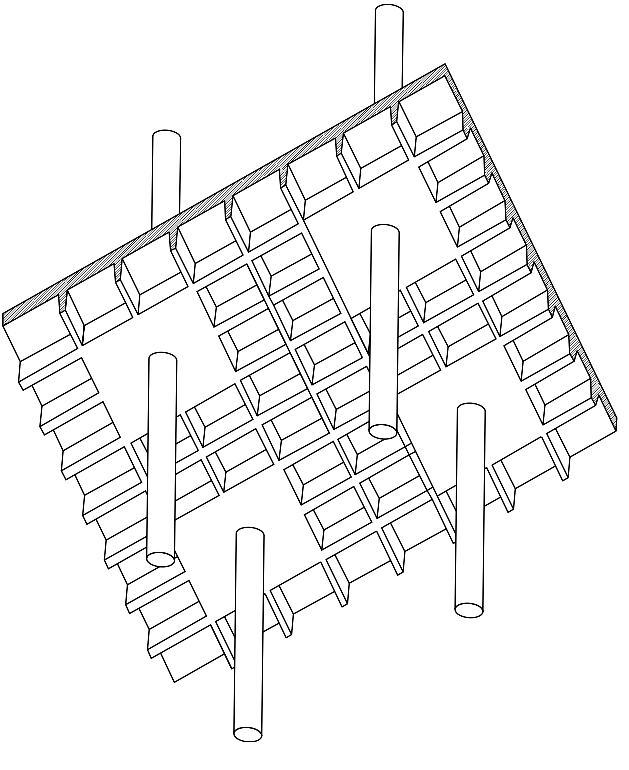

FIGURE 7.20 Waffle slab.

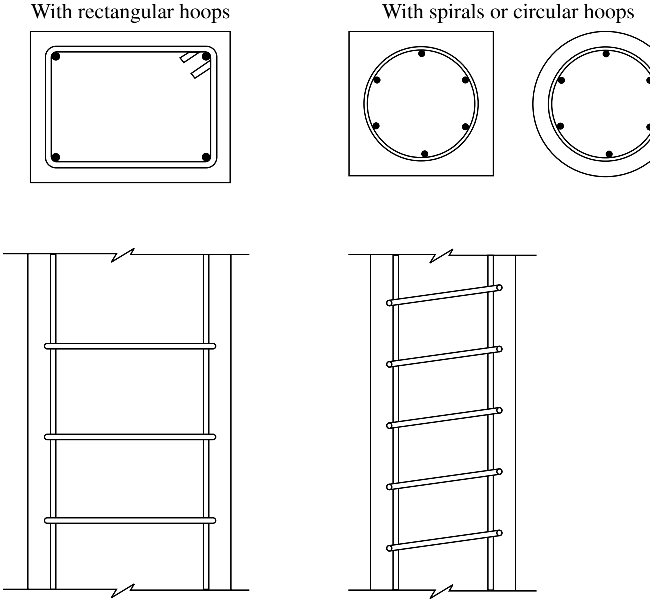

FIGURE 7.21 Typical reinforced concrete columns.

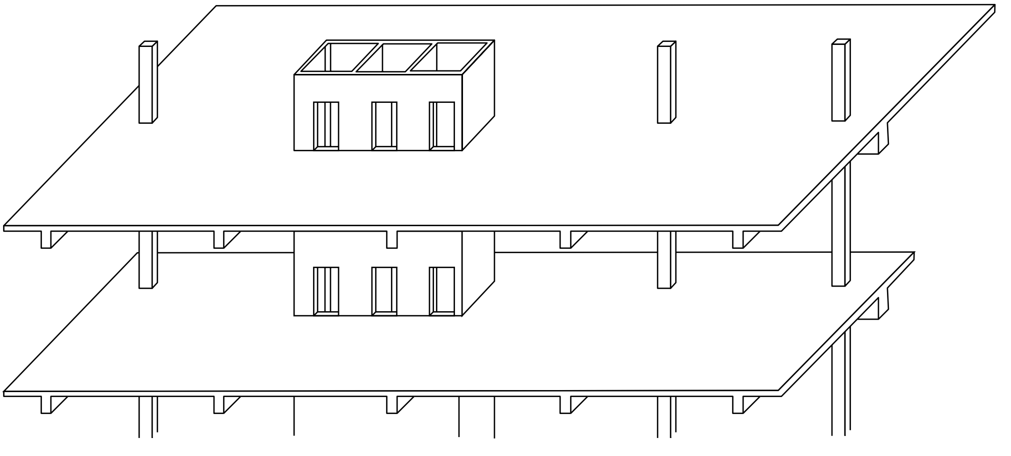

FIGURE 7.22 Reinforced concrete building elements.

7.14.1 Capacity of Columns under Pure Compression

Under pure compression (i.e., no moment) the axial capacity of columns reinforced with hoops and ties as transverse reinforcement is the sum of the axial capacity of the concrete and the steel:

$$\phi P_{n,max} = \phi\phi_{ecc}[0.85f'_c(A_g - A_{st}) + A_{st}f_y] \tag{7.38}$$

The strength reduction factor ϕ for tied columns is 0.65. The additional reduction factor ϕ_{ecc} shown in the equation accounts for accidental eccentricity from loading or due to construction tolerances that will induce moment. For tied column $\phi_{ecc} = 0.80$. For spiral columns, $\phi = 0.75$ and $\phi_{ecc} = 0.85$. Columns reinforced with spiral reinforcement are more ductile and reliable in sustaining axial load after spalling of concrete cover. Hence, lower reduction factors are assigned by ACI.

7.14.2 Preliminary Sizing of Columns

For columns that are expected to carry no or low moment, the previous equation can be rearranged to estimate the required gross cross-sectional area to resist the axial force demand P_u:

$$A_g > \frac{(P_u/\phi\phi_{ecc}) - A_{st}f_y}{0.85f_c'} \tag{7.39}$$

The ACI Code limits the column reinforcement area A_{st} to 1 to 8% of A_g. Reinforcement percentages less than 4% are usually more practical in terms of avoiding congestion and to ease fabrication. If a column is expected to carry significant moment, the A_g estimated by the above expression would not be adequate. To obtain an initial trial size in that case, the above A_g estimate may be increased by an appropriate factor (e.g., doubling or more).

7.14.3 Capacity of Columns under Combined Axial Force and Moment

Under the combined actions of axial force and moment, the capacity envelope of a column is generally described by an interaction diagram (see Figure 7.23). Load demand points (M_u, P_u) from all load combinations must fall inside the $\phi P_n - \phi M_n$ capacity envelope; otherwise, the column is considered inadequate and should be redesigned. Computer software are typically used in design practice to generate column interaction diagrams.

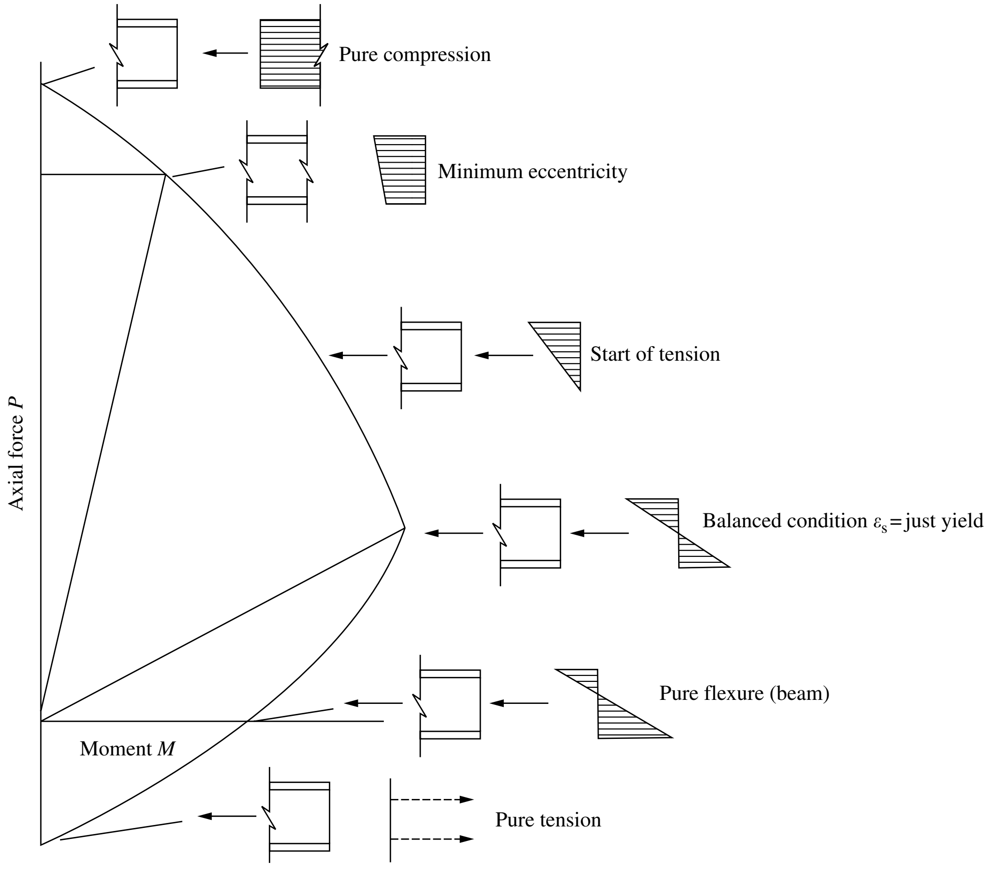

FIGURE 7.23 Column interaction diagram.

The upper point of an interaction curve is the case of pure axial compression. The lowest point is the case of pure axial tension, $\phi P_{\mathrm{n,tension}} = \phi A_{\mathrm{st}} f_{\mathrm{y}}$ (it is assumed that the concrete section cracks and supplies no tensile strength). Where the interaction curve intersects with the moment axis, the column is under pure bending, in which case the column behaves like a beam. The point of maximum moment on the interaction diagram coincides with the balanced condition. The extreme concrete fiber strain reaches ultimate strain (0.003) simultaneously with yielding of the extreme layer of steel on the opposite side ($f_{\mathrm{y}}/E_{\mathrm{s}} = 0.002$).

Each point of the column interaction curve represents a unique strain distribution across the column section. The axial force and moment capacity at each point is determined by a strain compatibility analysis, similar to that presented for beams (see Section 7.12) but with an additional axial force component. The strain at each steel level i is obtained from similar triangles $\varepsilon_{\mathrm{s}i} = 0.003(c - d_i)/c$. Then, the steel stress at each level is $f_{\mathrm{st}} = \varepsilon_{\mathrm{s}i} E_{\mathrm{s}}$, but not greater in magnitude than the yield stress f_{y}. The steel force at each level is computed by $F_{\mathrm{s}i} = A_{\mathrm{s}i} f_{\mathrm{s}i}$. The depth of the equivalent concrete compressive stress block a is approximated by the relationship $a = \beta_1 c$. β_1 is the concrete stress block factor given in Figure 7.5. Hence, the resultant concrete compression force may be expressed as $C_{\mathrm{c}} = 0.85 f_{\mathrm{c}}' ab$. To satisfy equilibrium, summing forces of the concrete compression and the n levels of the steel, the axial capacity is obtained as

$$\phi P_{\mathrm{n}} = \phi\left(C_{\mathrm{c}} + \sum_{i=1}^{n} F_{\mathrm{s}i}\right) \tag{7.40}$$

The flexural capacity is obtained from summation of moments about the plastic centroid of the column

$$\phi M_{\mathrm{n}} = \phi\left[C_{\mathrm{c}}\left(\frac{h}{2} - \frac{a}{2}\right) + \sum_{i=1}^{n} F_{\mathrm{s}i}\left(\frac{h}{2} - d_i\right)\right] \tag{7.41}$$

The strength reduction ϕ factor is not a constant value over the column interaction curve. For points above the balanced point ϕ is 0.65 for tied columns and 0.70 for spiral columns. In this region the column section is compression controlled (extreme level steel strain is at or below yield) and has less ductility. Below the balanced point the column section becomes tension controlled (extreme steel strain greater than yield) and the behavior is more ductile, hence ϕ is allowed to increase linearly to 0.90. This transition occurs between the balanced point and where the extreme steel strain is at 0.005.

7.14.4 Detailing of Column Longitudinal Reinforcement

Longitudinal bars in a column are generally detailed to run continuous by through the story height without cutoffs. In nonseismic regions, column bars are generally spliced above the floor slab to ease construction. In seismic design, column splice should be located at midstory height, away from the section of maximum stress. See Section 7.17 on column splice lengths.

Where the column cross-section dimensions change, longitudinal bars need to be offset. The slope of the offset bar should not exceed 1 in 6. Horizontal ties are needed within the offset to resist 1.5 times the horizontal component of the offset bars. Offsets bents are not allowed if the column face is offset by 3 in. or more.

7.14.5 Shear Design of Columns

The general shear design procedure for selecting transverse reinforcement for columns is similar to that for beams (see Section 7.12). In columns, the axial compression load N_{u} enhances the concrete shear strength, hence, in lieu of the simplified $V_{\mathrm{c}} = 2\sqrt{f_{\mathrm{c}}'} b_{\mathrm{w}} d$, alternative formulas may be used:

$$V_{\mathrm{c}} = 2\left(1 + \frac{N_{\mathrm{u}}}{2000 A_{\mathrm{g}}}\right)\sqrt{f_{\mathrm{c}}'} b_{\mathrm{w}} d \tag{7.42a}$$

The quantity N_u/A_g must be in units of pounds per square inch. A second alternative formula for concrete shear strength V_c is

$$V_c = \left(1.9\sqrt{f_c'} + 2500\rho_w \frac{V_u d}{M_m}\right) b_w d \leq 3.5\sqrt{f_c'} b_w d \sqrt{1 + \frac{N_u}{500A_g}} \quad (7.42b)$$

where

$$M_m = M_u - N_u \frac{(4h - d)}{8} \quad (7.43)$$

If M_m is negative, the upper bound expression for V_c is used.

Under seismic conditions, additional transverse reinforcement is required to confine the concrete to enhance ductile behavior. See Section IV of this book on earthquake design.

7.14.6 Detailing of Column Hoops and Ties

The main transverse reinforcement should consist of one or a series of perimeter hoops (see Figure 7.24), which not only serve as shear reinforcement, but also prevent the longitudinal bars from buckling out through the concrete cover. Every corner and alternate longitudinal bar should have a hook support (see Figure 7.24). The angle of the hook must be less than 135°. All bars should be hook supported if the clear spacing between longitudinal bars is more than 6 in. The transverse reinforcement must be at least a No. 3 size if the longitudinal bars are No. 10 or smaller, and at least a No. 4 size if the longitudinal bars are greater than No. 10.

To prevent buckling of longitudinal bars, the vertical spacing of transverse reinforcement in columns should not exceed 16 longitudinal bar diameters, 48 transverse bar diameters, or the least dimension of the column size.

7.14.7 Design of Spiral Columns

Columns reinforced with spirals provide superior confinement for the concrete core. Tests have shown that spiral columns are able to carry their axial load even after spalling of the concrete cover. Adequate confinement is achieved when the center-to-center spacing s of the spiral of diameter d_b and yield strength f_y satisfies the following:

$$s \leq \frac{\pi f_y d_b^2}{0.45 h_c f_c' \left[\left(A_g/A_c\right) - 1\right]} \quad (7.44)$$

where h_c is the diameter of the concrete core measured out-to-out of the spiral.

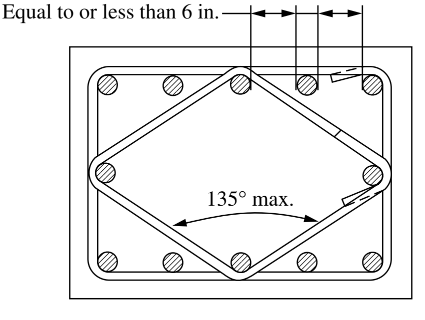

FIGURE 7.24 Column transverse reinforcement detailing.

7.14.8 Detailing of Columns Spirals

Spiral columns require a minimum of six longitudinal bars. Spacers should be used to maintain the design spiral spacing and to prevent distortions. The diameter of the spiral d_b should not be less than $\frac{3}{8}$ in. The clear spacing between spirals should not exceed 3 in. or be less than 1 in. Spirals should be anchored at each column end by providing an extra one and one-half turns of spiral bar. Spirals may be spliced by full mechanical or welded splices or by lap splices with lap lengths not less than 12 in. or $45d_b$ ($72d_b$ if plain bar). While spirals are not required to run through the column-to-floor connection zones, ties should be inserted in those zones to maintain proper confinement, especially if horizontal beams do not frame into these zones.

7.14.9 Detailing of Column to Beam Joints

Joints will perform well if they are well confined. By containing the joint concrete, its structural integrity is ensured under cyclic loading, which allows the internal force capacities, as well as the splices and anchorages detailed within the joint, to develop. Often, confinement around a joint will be provided by the beams or other structural elements that intersect at the joint, if they are of sufficient size. Otherwise, some closed ties, spirals, or stirrups should be provided within the joint to confine the concrete. For nonseismic design, the ACI has no specific requirements on joint confinement.

7.14.10 Columns Subject to Biaxial Bending

If a column is subject to significant moments biaxially, for example, a corner column at the perimeter of a building, the column capacity may be defined by an interaction surface. This surface is essentially an extension of the 2-D interaction diagram described in Figure 7.23 to three coordinate axes $\phi P_n - \phi M_{nx} - \phi M_{ny}$. For rectangular sections under biaxial bending the resultant moment axis may not coincide with the neutral axis. (This is never the case for a circular cross-section because of point symmetry.) An iterative procedure is necessary to determine this angle of deviation. Hence, an accurate generation of the biaxial interaction surface generally requires computer software. Other approximate methods have been proposed. The ACI Code Commentary (R10.3.7) presents the Reciprocal Load Method in which the biaxial capacity of a column ϕP_{ni} is related in a reciprocal manner to its uniaxial capacities, ϕP_{nx} and ϕP_{ny}, and pure axial capacity P_0:

$$\frac{1}{\phi P_{ni}} = \frac{1}{\phi P_{nx}} + \frac{1}{\phi P_{ny}} - \frac{1}{\phi P_0} \tag{7.45}$$

7.14.11 Slender Columns

When columns are slender the internal forces determined by a first-order analysis may not be sufficiently accurate. The change in column geometry from its deflection causes secondary moments to be induced by the column axial force, also referred to as the P–Δ effect. In stocky columns these secondary moments are minor. For columns that are part of a nonsway frame, for which analysis shows limited side-sway deflection, the effects of column slenderness can be neglected if the column slenderness ratio

$$\frac{kl_u}{r} \leq 34 - 12(M_1/M_2) \tag{7.46}$$

The effective length factor k can be obtained from Figure 7.25 or be conservatively assumed to be 1.0 for nonsway frames. The radius of gyration r may be taken to be 0.30 times the overall dimension of a rectangular column (in the direction of stability) or 0.25 times the diameter for circular columns. The ratio of the column end moments (M_1/M_2) is taken as positive if the column is bent in single curvature, and negative in double curvature.

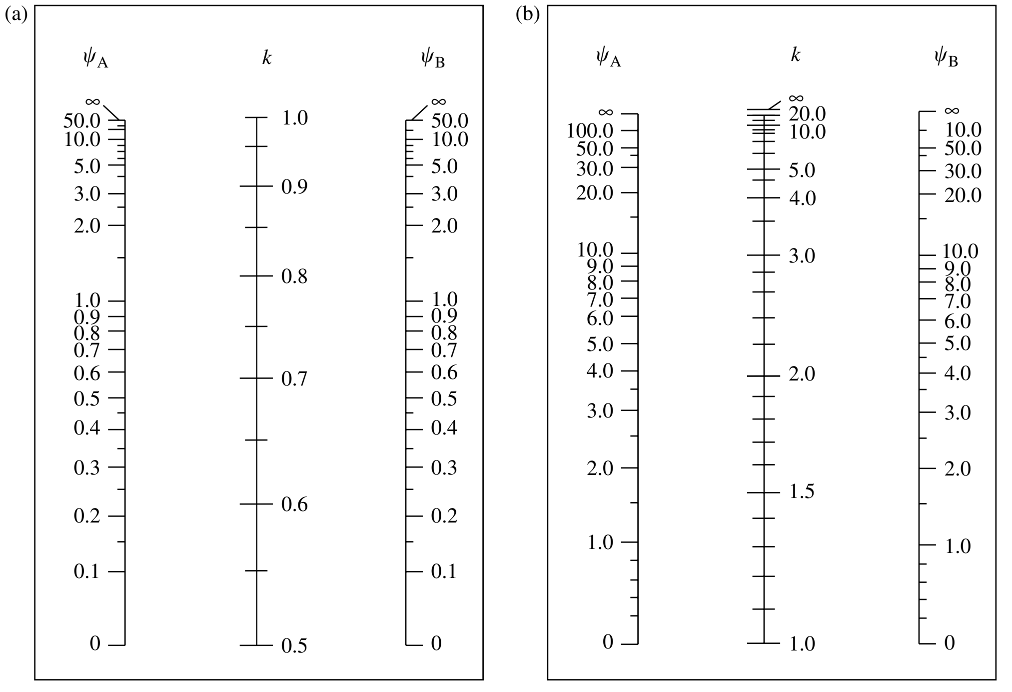

FIGURE 7.25 Effective length factor k: (a) nonsway frames and (b) sway frames.
Note: ψ is the ratio of the summation of column stiffness $[\sum(EI/L)]$ to beam stiffness at the beam–column joint.

For a building story, a frame is considered to be nonsway if its stability index

$$Q = \frac{\sum P_u \Delta_0}{V_u l_c} \leq 0.05 \tag{7.47}$$

where Δ_0 is the first-order relative deflection between the top and bottom of the story and $\sum P_u$ and V_u are the total vertical load and story shear, respectively.

For sway frames, slenderness may be neglected if the slenderness ratio $kl_u/r \leq 22$. The k factor must be taken as greater than or equal to 1.0 (see Figure 7.25).

For structural design, it is preferable to design reinforced concrete structures as nonsway systems and with stocky columns. Structural systems should be configured with stiff lateral resistant elements such as shear walls to control sway. Column cross-sectional dimensions should be selected with the slenderness criteria in mind.

If slender columns do exist in a design, adopting a computerized second-order analysis should be considered so that the effects of slenderness will be resolved internally by the structural analysis (see Section 7.7). Then, the internal force demands from the computer output can be directly checked against the interaction diagram in like manner as a nonslender column design. Alternatively, the ACI code provides a manual method called the Moment Magnifier Method to adjust the structural analysis results of a first-order analysis.

7.14.12 Moment Magnifier Method

The Moment Magnifier Method estimates the column moment M_c in a slender column by magnifying the moment obtained from a first-order analysis M_2. For the nonsway case, the factor δ_{ns} magnifies the column moment:

$$M_c = \delta_{ns} M_2 \tag{7.48}$$

where

$$\delta_{ns} = \frac{C_m}{1 - (P_u/0.75P_c)} \geq 1.0 \tag{7.49}$$

and

$$P_c = \frac{\pi^2 EI}{(kl_u)^2} \tag{7.50}$$

The column stiffness may be estimated as

$$EI = \frac{(0.20E_c I_g + E_s I_{se})}{1 + \beta_d} \tag{7.51}$$

or a more simplified expression may be used:

$$EI = \frac{0.4E_c I_g}{1 + \beta_d} \tag{7.52}$$

In the sway case, the nonsway moments M_{ns} (e.g., gravity loads) are separated from the sway moments M_s (e.g., due to wind, unbalanced live loads). Only the sway moment is magnified:

$$M_c = M_{ns} + \delta_s M_s \tag{7.53}$$

$$\delta_s M_s = \frac{M_s}{1 - Q} \geq M_s \tag{7.54}$$

where Q is the stability index given by Equation 7.47.

7.15 Walls

If tall walls (or shear walls) and combined walls (or core walls) subjected to axial load and bending behave like a column, the design procedures and formulas presented in the previous sections are generally applicable. The reinforcement detailing of wall differs from that of columns. Boundary elements, as shown in Figure 7.26, may be attached to the wall ends or corners to enhance moment capacity. The ratio ρ_n of vertical shear reinforcement to gross area of concrete of horizontal section should not be less than

$$\rho_n = 0.0025 + 0.5\left(2.5 - \frac{h_w}{l_w}\right)(\rho_h - 0.0025) \geq 0.0025 \tag{7.55}$$

The spacing of vertical wall reinforcement should not exceed $l_w/3$, $3h$, or 18 in. To prevent buckling, the vertical bars opposite each other should be tied together with lateral ties if the vertical reinforcement is greater than 0.01 the gross concrete area.

7.15.1 Shear Design of Walls

The general shear design procedure given in Section 7.12 for determining shear reinforcement in columns applies to walls. For walls in compression, the shear strength provided by concrete V_c may be taken as $2\sqrt{f'_c}hd$. Alternatively, V_c may be taken from the lesser of

$$3.3\sqrt{f'_c}hd + \frac{N_u d}{4l_w} \tag{7.56}$$

and

$$\left[0.6\sqrt{f'_c} + \frac{l_w(1.25\sqrt{f'_c} + 0.2(N_u/l_w h))}{(M_u/V_u) - (l_w/2)}\right]hd \tag{7.57}$$

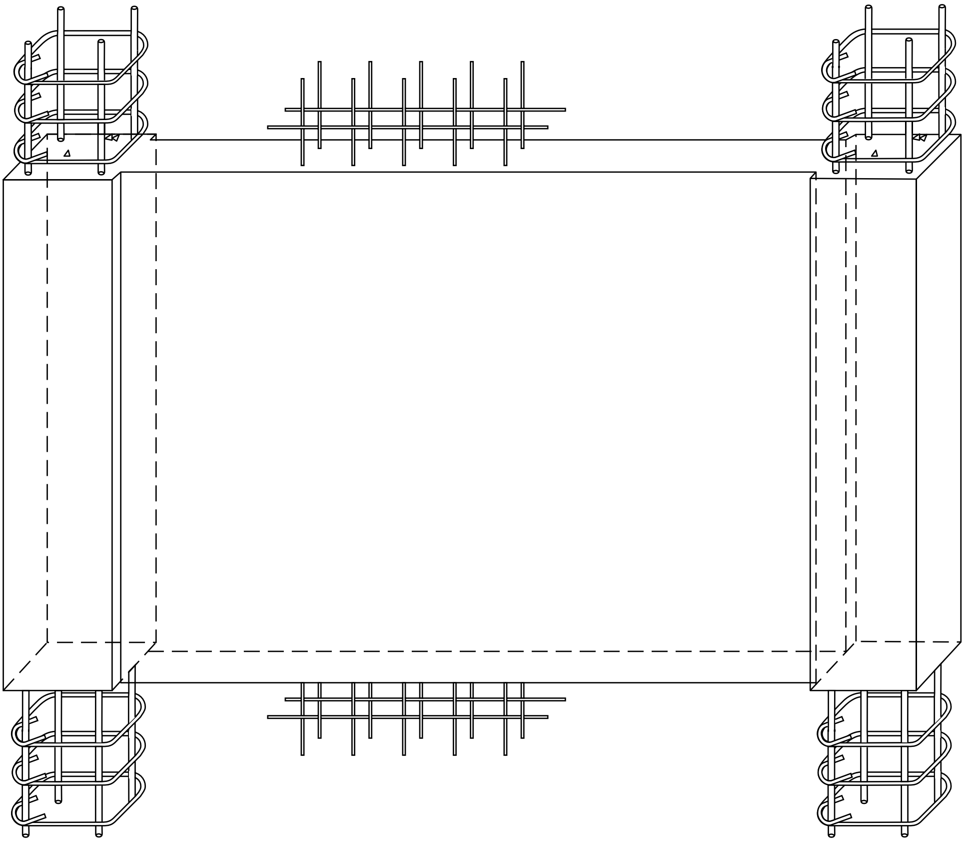

FIGURE 7.26 Reinforced concrete wall with boundary columns.

In lieu of a strain compatibility analysis, the depth of walls d may be assumed to be $0.8l_w$. Shear strength provided by the horizontal reinforcement in walls is also calculated by the equation $V_s = A_v f_y d/s$. The shear capacity of walls $\phi V_n = \phi(V_c + V_s)$ should not be greater than $\phi 10\sqrt{f'_c}hd$.

The spacing of horizontal wall reinforcement should not exceed $l_w/5$, $3h$, or 18 in. The minimum ratio of horizontal wall reinforcement should be more than 0.0025 (or 0.0020 for bars not larger than No. 5). The vertical and horizontal wall bars should be placed as close to the two faces of the wall as cover allows.

7.16 Torsion Design

Torsion will generally not be a serious design issue for reinforced concrete structures if the structural scheme is regular and symmetrical in layout and uses reasonable member sizes. In building floors, torsion may need to be considered for edge beams and members that sustain large unbalanced loading. Concrete members are relatively tolerant of torsion. The ACI permits torsion design to be neglected if the factored torsional moment demand T_u is less than

$$\phi\sqrt{f'_c}\left(\frac{A_{cp}^2}{P_{cp}}\right) \tag{7.58}$$

which corresponds to about one quarter of the torsional cracking capacity. For hollow sections the gross area of section A_g should be used in place of A_{cp}. If an axial compressive or tensile force N_u exists, the torsion design limit becomes

$$\phi\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f_c'}}} \quad (7.59)$$

If the torsional moment demands are higher than the above limits, the redistribution of torque after cracking may be taken into account, which occurs if the member is part of an indeterminate structural system. Hence, in torsion design calculations, the torsional moment demand T_u need not be taken greater than

$$\phi 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right) \quad (7.60)$$

If axial force is present, the upper bound on the design torque T_u is

$$\phi 4\sqrt{f_c'}\left(\frac{A_{cp}^2}{p_{cp}}\right)\sqrt{1+\frac{N_u}{4A_g\sqrt{f_c'}}} \quad (7.61)$$

7.16.1 Design of Torsional Reinforcement

The torsional moment capacity may be based on the space truss analogy (see Figure 7.27). The space truss formed by the transverse and longitudinal reinforcement forms a mechanism that resists torsion. To be effective under torsion, the transverse reinforcement must be constructed of closed hoops (or closed ties) perpendicular to the axis of the member. Spiral reinforcement or welded wire fabric may be used.

To prevent failure of the space truss from concrete crushing and to control diagonal crack widths, the cross-section dimensions must be selected to satisfy the following criteria. For solid sections

$$\sqrt{\left(\frac{V_u}{b_w d}\right)^2+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right)^2} \le \phi\left(\frac{V_c}{b_w d}+8\sqrt{f_c'}\right) \quad (7.62)$$

and for hollow sections

$$\left(\frac{V_u}{b_w d}\right)+\left(\frac{T_u p_h}{1.7A_{oh}^2}\right) \le \phi\left(\frac{V_c}{b_w d}+8\sqrt{f_c'}\right) \quad (7.63)$$

After satisfying these criteria, the torsional moment capacity is determined by

$$\phi T_n = \phi\frac{2A_o A_t f_{yv}}{s}\cot\theta \quad (7.64)$$

The shear flow area A_o may be taken as $0.85A_{oh}$, where A_{oh} is the area enclosed by the closed hoop (see Figure 7.28). The angle θ may be assumed to be 45°. More accurate values of A_o and θ may be used from analysis of the space truss analogy.

To determine the additional transverse torsional reinforcement required to satisfy ultimate strength, that is, $\phi T_n \ge T_u$, the transverse reinforcement area A_t and its spacing s must satisfy the following:

$$\frac{A_t}{s} > \frac{T_u}{\phi 2A_o f_{yv}\cot\theta} \quad (7.65)$$

The area A_t is for one leg of reinforcement. This torsional reinforcement area should then be combined with the transverse reinforcement required for shear demand A_v (see Section 7.11). The total transverse reinforcement required for the member is thus

$$\frac{A_v}{s}+2\frac{A_t}{s} \quad (7.66)$$

The above expression assumes that the shear reinforcement consists of two legs. If more than two legs are present, only the legs adjacent to the sides of the cross-section are considered effective for torsional resistance.

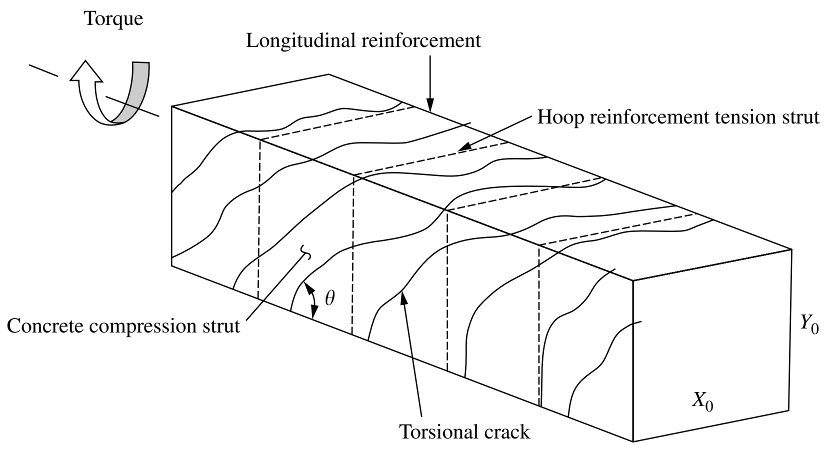

FIGURE 7.27 Truss analogy for torsion.

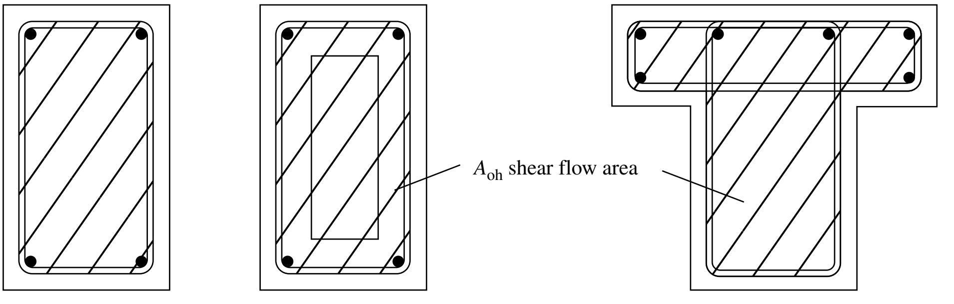

FIGURE 7.28 Torsional reinforcement and shear flow area.

The total transverse reinforcement must exceed the following minimum amounts:

$$0.75\sqrt{f_c'}\frac{b_w}{f_{yv}} \geq \frac{50b_w}{f_{yv}} \tag{7.67}$$

A minimum amount of longitudinal reinforcement is also required:

$$A_l = \frac{A_t}{s} p_h \left(\frac{f_{yv}}{f_{yt}}\right) \cot^2\theta \tag{7.68}$$

The reinforcement area A_l is additional to that required for resisting flexure and axial forces and should not be less than

$$\frac{5\sqrt{f_c'}A_{cp}}{f_{yl}} - \left(\frac{A_t}{s}\right) p_h \frac{f_{yv}}{f_{yl}} \tag{7.69}$$

where A_t/s should not be less than $25b_w/f_{yv}$. The torsional–longitudinal reinforcement should be distributed around the section in a uniform manner.

7.16.2 Detailing of Torsional Reinforcement

The spacing of closed transverse reinforcement under torsion must not exceed $p_h/8$ or 12 in. Torsion reinforcement should be provided for a distance of at least $(b_t + d)$ beyond the point theoretically required. Torsional stresses cause unrestrained corners of the concrete to spall off. Transverse torsion reinforcement needs to be anchored by 135° hooks. In hollow cross-sections, the closed hoops should be placed near the outer surface of the wall. The distance from the centerline of the hoop reinforcement to the inside wall face should not be less than $0.5A_{oh}/p_h$.

The longitudinal torsion reinforcement should be distributed so that its centroid is near the centroid of the cross-section. It should be distributed around the perimeter and be positioned inside the closed hoop with a maximum spacing of 12 in. There should be at least one longitudinal bar at each corner of the hoop. The longitudinal reinforcement must have a diameter of at least 0.042 times the hoop spacing. The ends of the longitudinal reinforcement must be fully developed for yielding. It is permitted to reduce the area of the longitudinal reinforcement by an amount equal to $M_u/(0.9df_{yl})$ since flexural compression offsets the longitudinal tension due to torsion.

7.17 Reinforcement Development Lengths, Hooks, and Splices

The various ultimate capacity formulas presented in the previous sections are premised on the assumption that the reinforcement will reach its yield strength f_y. This is not assured unless the reinforcement has (1) sufficient straight embedment length on each side of the point of yielding, (2) a hook of sufficient anchorage capacity, or (3) a qualified mechanical anchor device.

7.17.1 Tension Development Lengths

The ACI development length equation for bars in tension l_d is expressed in terms of a multiple of the bar diameter d_b (inch unit):

$$l_d = \left(\frac{3}{40} \frac{f_y}{\sqrt{f'_c}} \frac{\alpha\beta\gamma\lambda}{(c + K_{tr})/d_b} \right) d_b \geq 12\,\text{in.} \tag{7.70}$$

where the transverse reinforcement index $K_{tr} = A_{tr}f_{yt}/1500sn$, which may be assumed to be zero for simplicity. Table 7.13 gives the development length for the case of normal weight concrete ($\lambda = 1.0$) and uncoated reinforcement ($\beta = 1.0$). Development lengths need to be increased under these conditions: beam reinforcement positioned near the top surface, epoxy coating, lightweight concrete, and bundling of bars (see ACI Section 12.2.4).

TABLE 7.13 Development Lengths in Tension

	Tension development length (in.)	
	Concrete strength (psi)	
Bar size	4000	8000
3	12	12
4	12	12
5	15	12
6	21	15
7	36	26
8	47	34
9	60	43
10	77	54
11	94	67
14	136	96
18	242	171

Note: Normal-weight concrete, Grade 60 reinforcement. $\alpha = 1.0$, $\beta = 1.0$, $c = 1.5$ in., and $K_{tr} = 0$.

7.17.2 Compression Development Lengths

For bars under compression, such as in columns, yielding is assured if the development length meets the largest value of $(0.02 f_y/\sqrt{f_c'})d_b$, $(0.0003 f_y)d_b$, and 8 in. Compression development lengths l_{dc} are given in Table 7.14. Compression development length may be reduced by the factor (A_s required)/(A_s provided) if reinforcement is provided in excess of that required by the load demand. Reinforcement within closely spaced spirals or tie reinforcement may be reduced by the factor 0.75 (spiral not less than $\frac{1}{4}$ in. in diameter and not more than 4 in. in pitch; column ties not less than No. 4 in size and spaced not more than 4 in.).

7.17.3 Standard Hooks

The standard (nonseismic) hook geometry as defined by ACI is shown in Figure 7.9. The required hook length l_{dh} is given in Table 7.15 and is based on the empirical formula $(0.02 f_y/\sqrt{f_c'})d_b$. Hook lengths may be reduced by 30% when the side and end covers over the hook exceed 2.5 and 2 in., respectively. A 20% reduction is permitted if the hook is within a confined concrete zone where the transverse

TABLE 7.14 Development Lengths in Compression

	Compression development length (in.)	
	Concrete strength (psi)	
Bar size	4000	8000
3	8	8
4	9	9
5	12	11
6	14	14
7	17	16
8	19	18
9	21	20
10	24	23
11	27	25
14	32	30
18	43	41

Note: Grade 60 reinforcement.

TABLE 7.15 Development Lengths of Hooks in Tension

	Development length of standard hook (in.)	
	Concrete strength (psi)	
Bar size	4000	8000
3	7	6
4	9	7
5	12	8
6	14	10
7	17	12
8	19	13
9	21	15
10	24	17
11	27	19
14	32	23
18	43	30

Note: Grade 60 steel. $\beta = 1.0$, $\lambda = 1.0$, l_{dh} not less than $8d_b$ nor 6 in.

reinforcement spacing is less than three times the diameter of the hooked bar. Note that whether the standard hook is detailed to engage over a longitudinal bar has no influence on the required hook length.

When insufficient hook length is available or in regions of heavy bar congestion, mechanical anchors may be used. There are a number of proprietary devices that have been tested and prequalified. These generally consist of an anchor plate attached to the bar end.

7.17.4 Splices

There are three choices for joining bars together: (1) mechanical device, (2) welding, and (3) lap splices. The mechanical and welded splices must be tested to show the development in tension or compression of at least 125% of the specified yield strength f_y of the bar. Welded splices must conform to ANSI/AWS D1.4, "Structural Welding Code — Reinforcing Steel." Since splices introduce weak leaks into the structure, they should be located as much as possible away from points of maximum force and critical locations.

7.17.4.1 Tension Lap Splices

Generally, bars in tension need to be lapped over a distance of $1.3l_d$ (Class B splice, see Section 7.17.1 for l_d), unless laps are staggered or more than twice the required steel is provided (Class A splice $= 1.0l_d$).

7.17.4.2 Compression Lap Splices and Column Splices

Compression lap splice lengths shall be $0.0005f_yd_b$, but not less than 12 in. If any of the load demand combinations is expected to introduce tension in the column reinforcement, column bars should be lapped as tension splices. Class A splices ($1.0l_d$) are allowed if half or fewer of the bars are spliced at any section and alternate lap splices are staggered by l_d. Column lap lengths may be multiplied by 0.83 if the ties provided through the lap splice length have an effective area not less than $0.0015hs$. Lap lengths within spiral reinforcement may be multiplied by 0.75.

7.18 Deflections

The estimation of deflections for reinforced concrete structures is complicated by the cracking of the concrete and the effects of creep and shrinkage. In lieu of carrying out a refined nonlinear analysis involving the moment curvature analysis of member sections, an elastic analysis may be used to incorporate a reduced or effective moment of inertia for the members. For beam elements an effective moment of inertia may be taken as

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \tag{7.71}$$

where the cracking moment of the section

$$M_{cr} = \frac{f_r I_g}{y_t} \tag{7.72}$$

The cracking stress or modulus of rupture of normal weight concrete is

$$f_r = 7.5\sqrt{f'_c} \tag{7.73}$$

For all-lightweight concrete f_r should be multiplied by 0.75, for sand-lightweight concrete, by 0.85.

For estimating the deflection of prismatic beams, it is generally satisfactory to take I_e at the section at midspan to represent the average stiffness for the whole member. For cantilevers, the I_e at the support should be taken. For nonprismatic beams, an average I_e of the positive and negative moment sections should be used.

Long-term deflections may be estimated by multiplying the immediate deflections of sustained loads (e.g., self-weight, permanent loads) by

$$\lambda = \frac{\xi}{1 + 50\rho'} \tag{7.74}$$

The time-dependent factor ξ is plotted in Figure 7.29. More refined creep and shrinkage deflection models are provided by ACI Committee 209 and the CEP-FIP Model Code (1990).

Deflections of beams and one-way slab systems must not exceed the limits in Table 7.16. Deflection control of two-way floor systems is generally satisfactory by following the minimum slab thickness

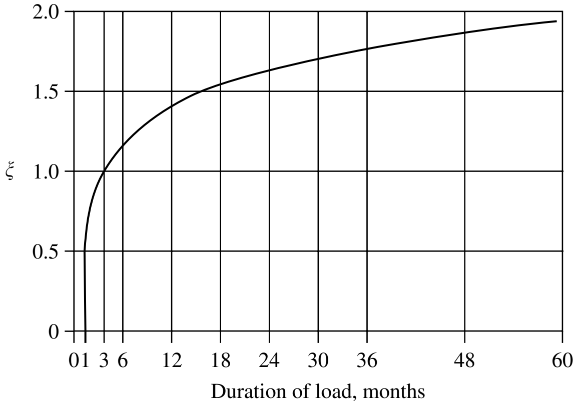

FIGURE 7.29 Time-dependent factor ξ.

TABLE 7.16 Deflection Limits of Beams and One-Way Slab Systems

Type of member	Deflection to be considered	Deflection limitation
Flat roots not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$l/180$[a]
Floors not supporting or attached to nonstructural elements likely to be damaged by large deflections	Immediate deflection due to live load L	$l/360$
Roof or floor construction supporting or attached to nonstructural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of nonstructural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)[b]	$l/480$[c]
Roof or floor construction supporting or attached to nonstructural elements not likely to be damaged by large deflections		$l/240$[d]

[a] Limit not intended to safeguard against ponding. Ponding should be checked by suitable calculations of deflection, including added deflections due to ponded water, and consideration of long-term effects of all sustained loads, camber, construction tolerances, and reliability of provisions for drainage.

[b] Long-term deflection should be determined in accordance with Equation 7.74, but may be reduced by the amount of deflection calculated to occur before attachment of nonstructural elements. This amount should be determined on the basis of accepted engineering date relating to time deflection characteristics of members similar to those being considered.

[c] Limit may be exceeded if adequate measures are taken to prevent to supported or attached elements.

[d] Limit should be greater than the tolerance provided for nonstructural elements. Limit may be exceeded if camber is provided so that total deflection minus camber does not exceed limit.

requirements (see Table 7.8). Lateral deflections of columns may be a function of occupancy comfort under high wind or seismic drift criteria (e.g., $H/200$).

7.19 Drawings, Specifications, and Construction

Although this chapter has focused mainly on the structural mechanics of design, design procedures and formulas, and rules that apply to reinforced concrete construction, the importance of drawings and specifications as part of the end products for communicating the structural design must not be overlooked. Essential information that should be included in the drawings and specifications are: specified compressive strength of concrete at stated ages (e.g., 28 days) or stage of construction; specified strength or grade of reinforced (e.g., Grade 60); governing design codes (e.g., IBC, AASHTO); live load and other essential loads; size and location of structural elements and locations; development lengths, hook lengths, and their locations; type and location of mechanical and welded splices; provisions for the effects of temperature, creep, and shrinkage; and details of joints and bearings.

The quality of the final structure is highly dependent on material and construction quality measures that improve durability, construction formwork, quality procedures, and inspection of construction. Although many of these aspects may not fall under the direct purview of the structural designer, attention and knowledge are necessary to help ensure a successful execution of the structural design. Information and guidance on these topics can be found in the *ACI Manual of Concrete Practice*, which is a comprehensive five-volume compendium of current ACI standards and committee reports: (1) Materials and General Properties of Concrete, (2) Construction Practices and Inspection, Pavements, (3) Use of Concrete in Buildings — Design, Specifications, and Related Topics, (4) Bridges, Substructures, Sanitary, and Other Special Structures, Structural Properties, and (5) Masonry, Precast Concrete, Special Processes.

Notation

a = depth of concrete stress block
A'_s = area of compression reinforcement
A_b = area of an individual reinforcement
A_c = area of core of spirally reinforced column measured to outside diameter of spiral
A_c = area of critical section
A_{cp} = area enclosed by outside perimeter of concrete cross-section
A_g = gross area of section
A_l = area of longitudinal reinforcement to resist torsion
A_o = gross area enclosed by shear flow path
A_{oh} = area enclosed by centerline of the outermost closed transverse torsional reinforcement
A_s = area of tension reinforcement
$A_{s,min}$ = minimum area of tension reinforcement
A_{st} = total area of longitudinal reinforcement
A_t = area of one leg of a closed stirrup resisting torsion within a distance s
A_{tr} = total cross-sectional area of all transverse reinforcement that is within the spacing s and that crosses the potential place of splitting through the reinforcement being developed
A_v = area of shear reinforcement
$A_{v,min}$ = minimum area of shear reinforcement
b = width of compression face
b_1 = width of critical section in l_1 direction
b_2 = width of critical section in l_2 direction
b_0 = perimeter length of critical section
b_t = width of that part of the cross-section containing the closed stirrups resisting torsion
b_w = web width
C = cross-sectional constant to define torsional properties $= \sum(1-0.63(x/y))/(x^3y/3)$ (total section is divided into separate rectangular parts, where x and y are the shorter and longer dimensions of each part, respectively).
c = distance from centroid of critical section to its perimeter (Section 7.13.2.1)
c = spacing or cover dimension
c_1 = dimension of column or capital support in l_1 direction

c_2 = dimension of column or capital support in l_2 direction
c_c = clear cover from the nearest surface in tension to the surface of the flexural reinforcement
C_c = resultant concrete compression force
C_m = factor relating actual moment diagram to an equivalent uniform moment
d = distance from extreme compression fiber to centroid of tension reinforcement
d' = distance from extreme compression fiber to centroid of compression reinforcement
d_b = nominal diameter of bar
d_i = distance from extreme compression fiber to centroid of reinforcement layer i
E_c = modulus of elasticity of concrete
E_{cb} = modulus of elasticity of beam concrete
E_{cs} = modulus of elasticity of slab concrete
EI = flexural stiffness of column
E_s = modulus of elasticity of steel reinforcement
f'_c = specified compressive strength of concrete
F_n = nominal structural strength
f_r = modulus of rupture of concrete
f_s = reinforcement stress
F_{si} = resultant steel force at bar layer i
f_y = specified yield stress of reinforcement
f_{yl} = specified yield strength of longitudinal torsional reinforcement
f_{yt} = specified yield strength of transverse reinforcement
f_{yv} = specified yield strength of closed transverse torsional reinforcement
h = overall thickness of column or wall
h_c = diameter of concrete core measured out-to-out of spiral
h_w = total height of wall
I_b = moment of inertia of gross section of beam
I_{cr} = moment of inertia of cracked section transformed to concrete
I_e = effective moment of inertia
I_s = moment of inertia of gross section of slab
I_{se} = moment of inertia of reinforcement about centroidal axis of cross-section
J_c = equivalent polar moment of inertia of critical section
k = effective length factor for columns
K_m = material constant
K_{tr} = transverse reinforcement index
L = member length
l_1 = center-to-center span length in the direction moments are being determined
l_2 = center-to-center span length transverse to l_1
l_c = center-to-center length of columns
l_d = development length of reinforcement in tension
l_{dc} = development length of reinforcement in compression
l_{dh} = development length of standard hook in tension, measured from critical section to outside end of hook
l_n = clear span length, measured from face-to-face of supports
l_u = unsupported length of columns
l_w = horizontal length of wall
M_1 = smaller factored end moment in a column, negative if bent in double curvature
M_2 = larger factored end moment in a column, negative if bent in double curvature
M_a = maximum moment applied for deflection computation
M_c = factored magnified moment in columns
M_{cr} = cracking moment
M_m = modified moment
M_n = nominal or theoretical moment strength
M_{ns} = factored end moment of column due to loads that do not cause appreciable side sway
M_0 = total factored static moment
M_s = factored end moment of column due to loads that cause appreciable side-ways
M_u = moment demand
M_{unb} = unbalanced moment at slab–column connections
n = modular ratio = E_s/E_c
N_C = resultant compressive force of concrete
N_T = resultant tensile force of reinforcement
N_u = factored axial load occurring simultaneously with V_u or T_u, positive sign for compression
P_c = critical load
p_{cp} = outside perimeter of concrete cross-section
p_h = perimeter of centerline of outermost concrete cross-section
P_n = nominal axial load strength of column

$P_{n,max}$ = maximum nominal axial load strength of column
P_{ni} = nominal biaxial load strength of column
P_{nx} = nominal axial load strength of column about x-axis
P_{ny} = nominal axial load strength of column about y-axis
P_0 = nominal axial load strength of column at zero eccentricity
P_u = axial load demand
Q = stability index
r = radius of gyration of cross-section
s = spacing of shear or torsional reinforcement along longitudinal axis of member
S_C = structural capacity
S_D = structural demand
T_n = nominal torsional moment strength
T_u = torsional moment demand
V_c = nominal shear strength provided by concrete
V_n = nominal shear strength
v_n = nominal shear stress strength of critical section
V_s = nominal shear strength provided by shear reinforcement
V_u = shear demand
v_u = shear stress at critical section
w_u = factored load on slab per unit area
y_t = distance from centroidal axis of gross section to extreme tension fiber
α = ratio of flexural stiffness of beam section to flexural stiffness of width of a slab bounded laterally by centerlines of adjacent panels on each side of beam = $E_{cb}I_b/E_{cs}I_s$
α = reinforcement location factor (Table 7.13)
α_i = angle between inclined shear reinforcement and longitudinal axis of member
α_m = average value of α for all beams on edges of a panel
α_s = shear strength factor
α_1 = α in direction of l_1
β = ratio of clear spans in long to short direction of two-way slabs
β = reinforcement coating factor (Section 7.17.1)
β_c = ratio of long side to short side dimension of column
β_d = ratio of maximum factored sustained axial load to maximum factored axial load
β_t = ratio of torsional stiffness of edge beam section to flexural stiffness of a width of slab equal to span length of beam, center-to-center of supports
β_1 = equivalent concrete stress block factor defined in Figure 7.5
δ_{ns} = nonsway column moment magnification factor
δ_s = sway column moment magnification factor
Δ_0 = first-order relative deflection between the top and bottom of a story
ε_c = concrete strain
ε_t = steel strain
γ = reinforcement size factor = 0.8 for No. 6 and smaller bars;
= 1.0 for No. 7 and larger
γ_f = fraction of unbalanced moment transferred by flexure at slab–column connections
γ_v = fraction of unbalanced moment transferred by eccentricity of shear at slab–column connections
λ = lightweight aggregate concrete factor (Section 7.17);
= 1.3 for light weight concrete
λ = multiplier for additional long-term deflection
ϕ_{ecc} = strength reduction factor for accidental eccentricity in columns
= 1.3 for lightweight concrete
ϕ_u = curvature at ultimate
ϕ_y = curvature at yield
ρ = ratio of tension reinforcement = A_s/bd
ρ' = ratio of compression reinforcement = A_s'/bd
ρ_h = ratio of horizontal wall reinforcement area to gross section area of horizontal section
ρ_n = ratio of vertical wall reinforcement area to gross section area of horizontal section
ρ_w = ratio of reinforcement = A_s/b_wd
ξ = time-dependent factor for sustained load
ϕ = strength reduction factor, see Table 7.4
θ = angle of compression diagonals in truss analogy for torsion

Useful Web Sites

American Concrete Institute: www.aci-int.org
Concrete Reinforcing Steel Institute: www.crsi.org
Portland Cement Association: www.portcement.org
International Federation of Concrete Structures: http://fib.epfl.ch
Eurocode 2: www.eurocode2.info
Reinforced Concrete Council: www.rcc-info.org.uk
Japan Concrete Institute: www.jci-net.or.jp
Emerging Construction Technologies: www.new-technologies.org